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DYNAMIC PROGRAMMING FOR OPTIMIZATION OF CAPACITOR ALLOCATION IN POWER DISTRIBUTION NETWORKS

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Albert Einstein & James Maxwell

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Summary

- Introduction
- Problem Formulation
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- How to solve it?
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Introduction

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As energy travels from generation plants to customers, electrical resistance in transmission and distribution lines causes dissipation of energy (*technical losses*).

Typically figures for these losses amount to around 7% of total energy production, 2% in transmission and 5% in distribution (according to ANEEL, technical losses in Brazilian distribution networks ranges from 2% to 18% with an average of 8%).

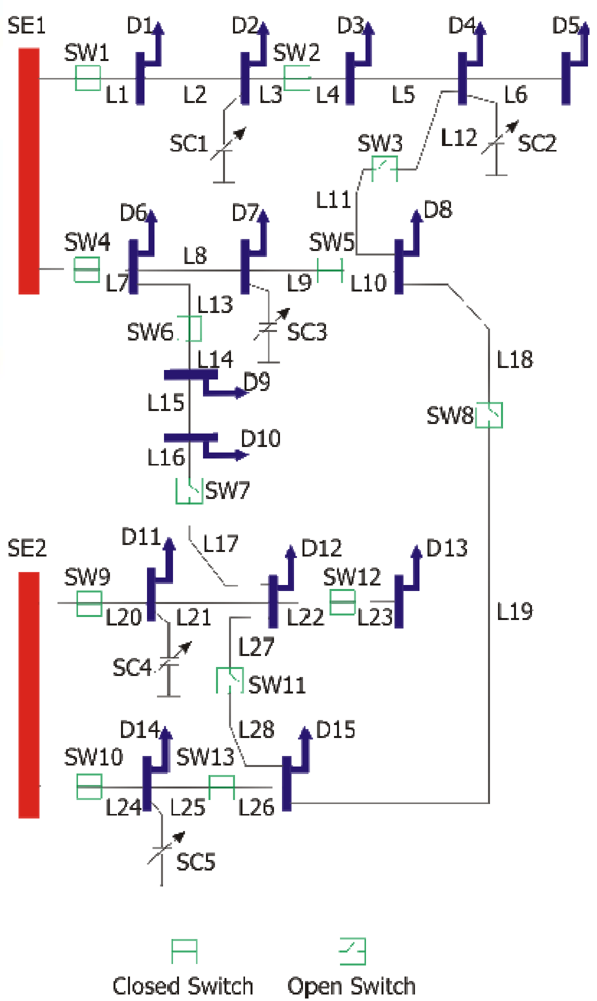
Loss reduction can be seen as a "hidden" source of energy.

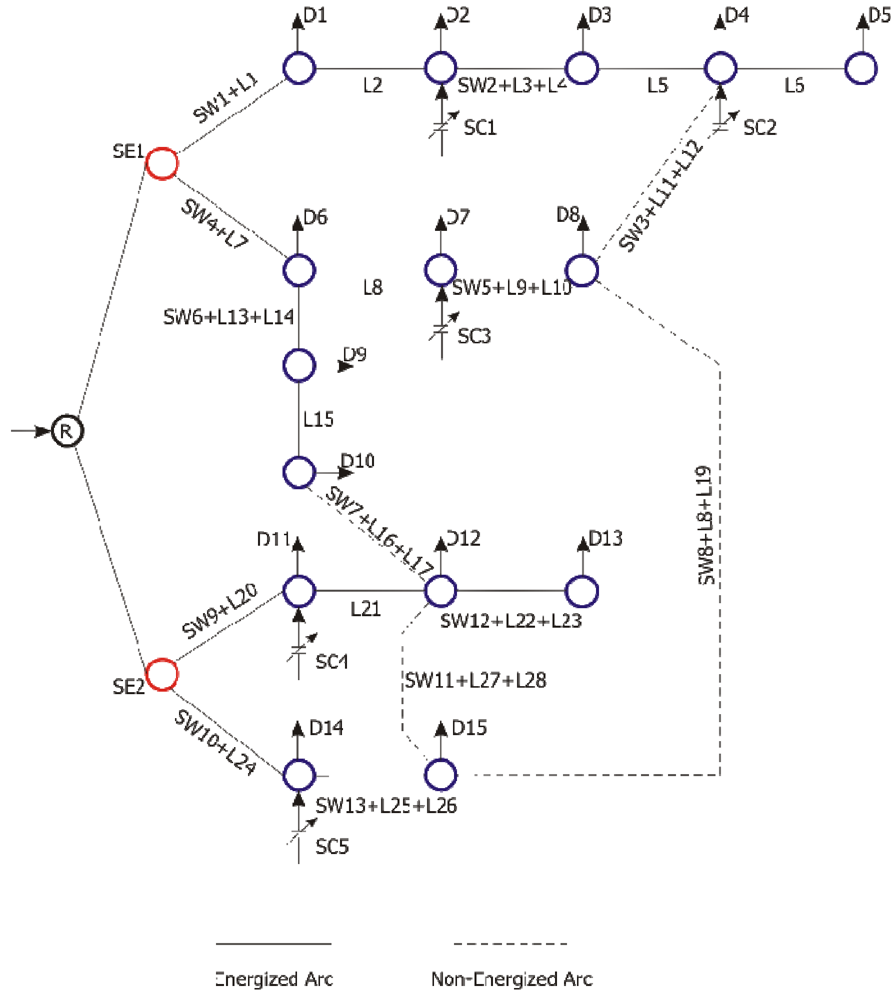
Some tools for loss reduction:

• Network reconfigurations;
• ***Capacitor bank allocation;***
• Improvements in cables and equipments.

Introduction

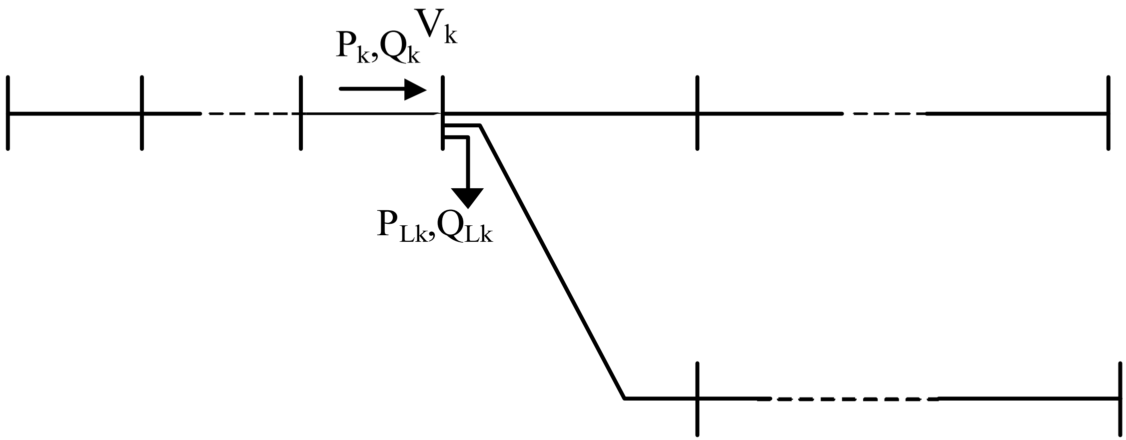
Main entities of a distribution network and its graph representation





In most of cases it operates with a radial configuration

A typical power distribution feeder with power flows in section *k*



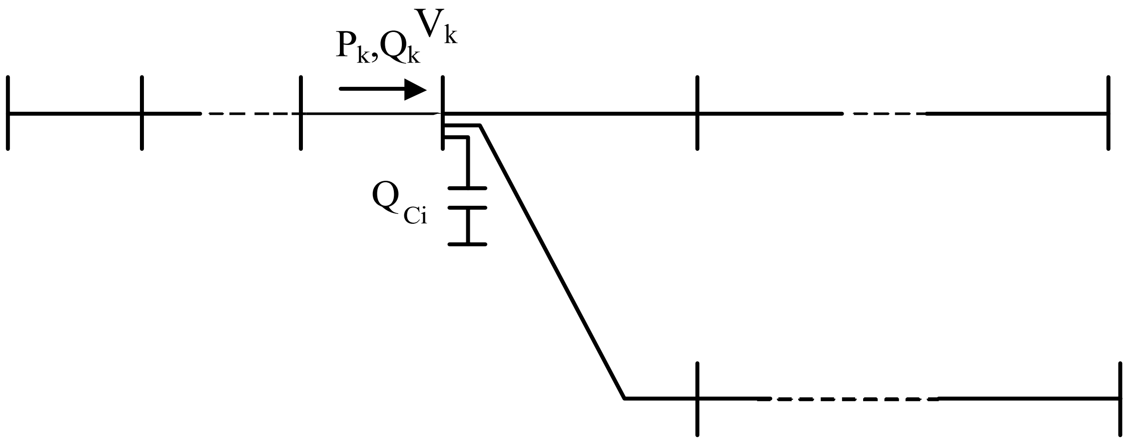
Technical losses (l_k) in a section *k:*

$$l_k = r_k(i_{Pk}^2) + r_k(i_{Qk}^2) = r_k\left(\frac{P_k^2 + Q_k^2}{V_k^2}\right)$$

i_{Pk} is the in-phase current component
i_{Qk} is the quadrature current component
r_k is the line resistance in section *k*
P_k is the active power (produces work)
Q_k is the reactive power

Introduction

Decreasing losses with capacitor banks



$$l_k = r_k(i_{Pk}^2) + r_k(i_{Qk}^2) = r_k\left(\frac{P_k^2 + (Q_k - Q_{ci})^2}{V_k^2}\right)$$

Q_{ci} is the reactive power injected at bus *k by capacitor* C_i

Capacitors can decrease the *reactive power* flowing back and forth in the network

Problem Formulation

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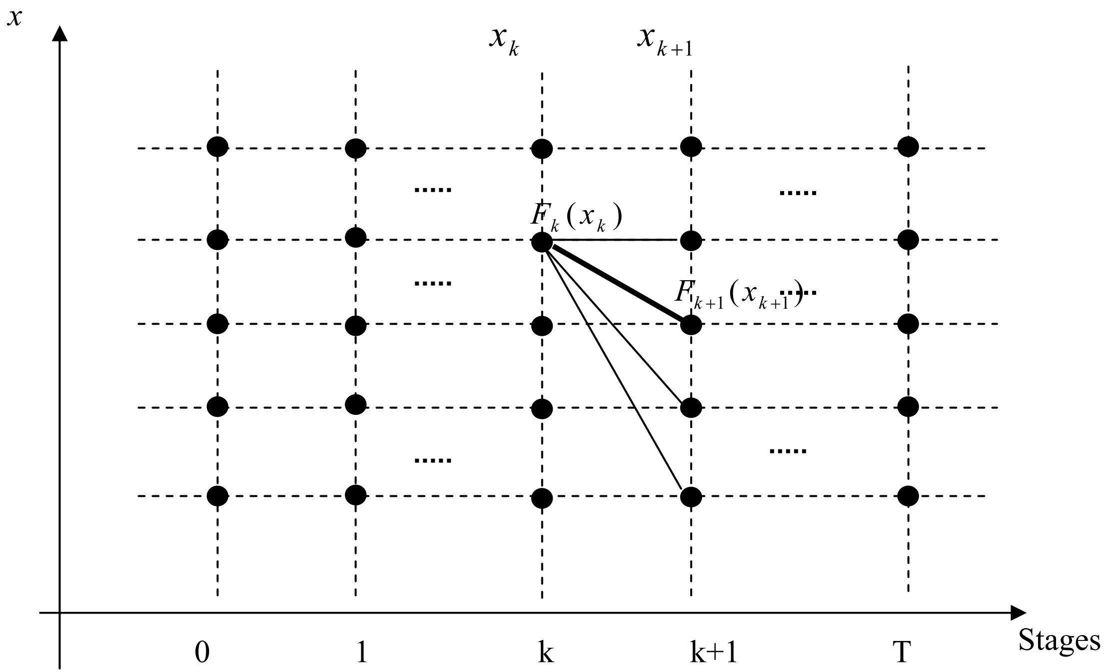
$$Min_{s \in S_C} \left\{ \sum_{i \in S_C} f(C_i) + \alpha_{et} \sum_{t \in T} \tau_t \sum_{k \in N} \sum_{j \in A_k} r_{kj} \frac{\left(P_{kj}\right)^2 + \left(Q_{kj}\right)^2}{V_k^2} \right\}$$

s. t:

- Active power flow equations
- Reactive power flow equations
- Voltage constraints

Durán´s DP Approach



$$F_k(x_k) = min_{u_k}\{\varphi_\mathrm{k}(x_k, u_k) + F_{k+1}(x_{k+1})\}$$

Durán´s DP Approach

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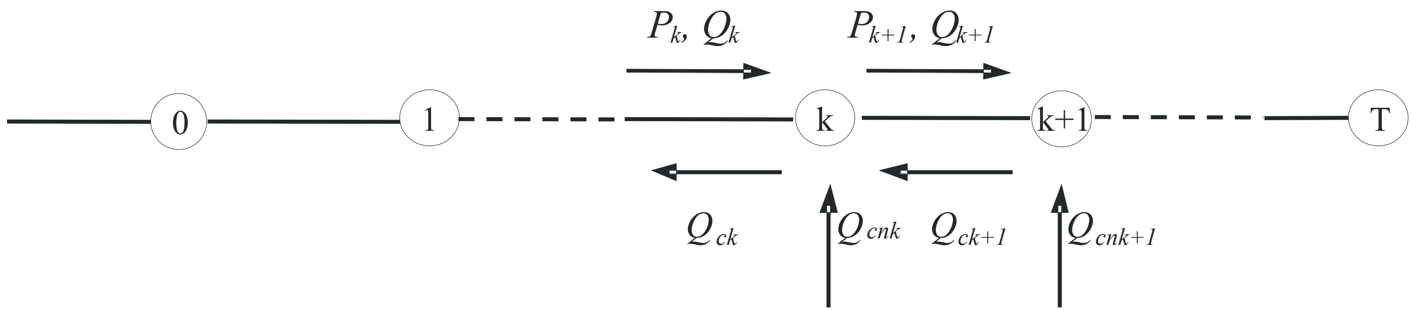
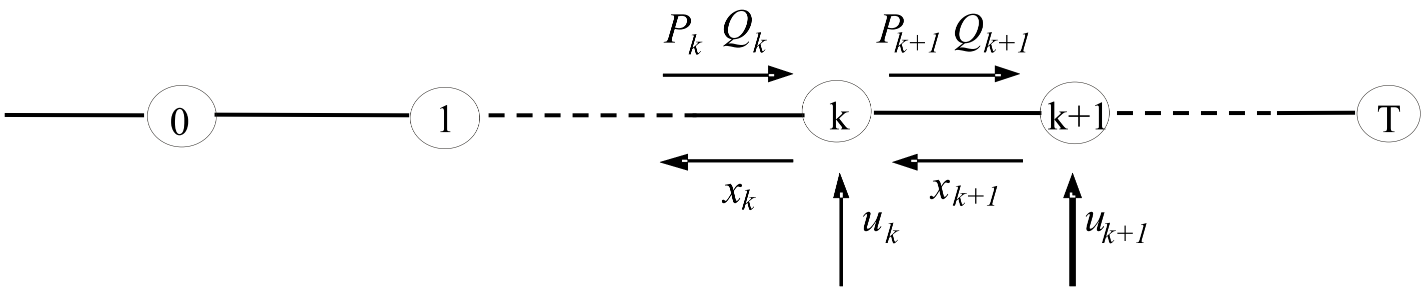
Durán (1968) proposed a DP approach to address the capacitor allocation problem in power distribution networks without *lateral branches*



- ***stages*** – all nodes in the power distribution network.
- ***control variable*** at a node *k* (u_k) - the capacitive reactive power (Q_{Ci}) injected at node *k.*
- ***state*** (x_k) – total capacitive power flowing upstream from node *k*.

A simple feeder with states and control variables at stages *k* and *k+1*



At stage *k:* $x_k = x_{k+1} + u_k$

If $V_k \cong 1.0\ p.u$ the total loss reduction in a section *k* is:

$$l_k^r = r_k\left(Q_k^2 - (Q_k - x_k)^2\right)$$

The economical value of the loss in section *k* in a given period of is:

$$c_k = \alpha_{et} l_k^r$$

The net benefit in section *k* is: $\varphi_k(x_k) = c_k - f(u_k)$

$f(u_k)$ is the cost of capacitor bank at node *k*.

Durán´s DP Approach

The optimization problem can be formulated as follows:

$$\max_u \left\{ \sum_{k \in N_I} \varphi_k(x_k) + \sum_{j \in N_F} \psi_j(x_j) \right\}$$

s.t:

$$x_{k+1} = x_k - u_k$$

$$\underline{u}_i \le u_i \le \overline{u}_i$$

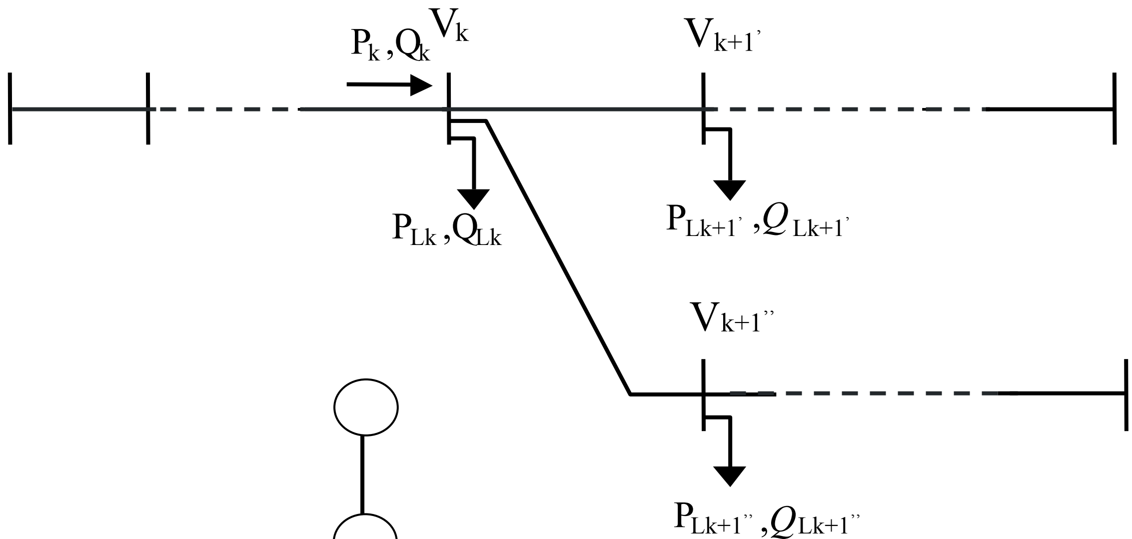
$$\underline{x}_i \le x_i \le \overline{x}_i$$

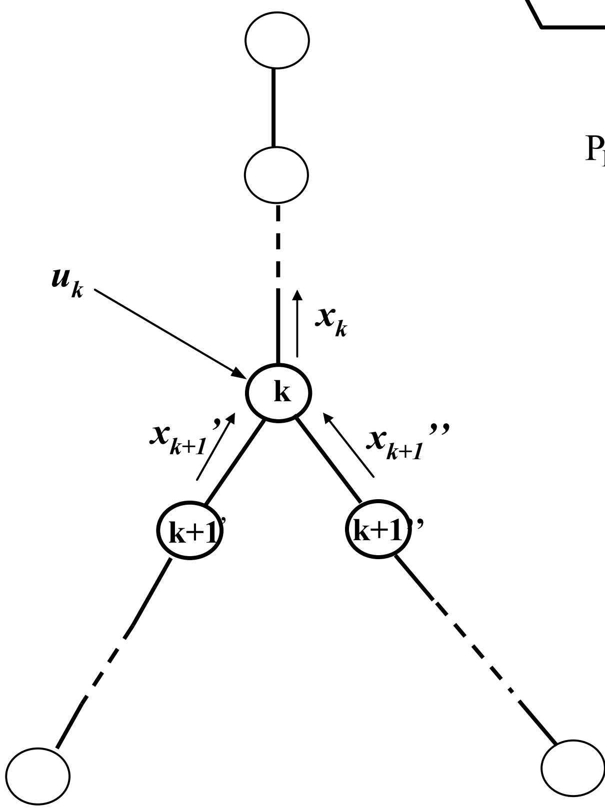
N_I: set of inner nodes.
N_F: set of leaf nodes.

$$\psi_j = c_j - f_j(x_j)$$

$$x_j = u_j$$

The New DP Approach

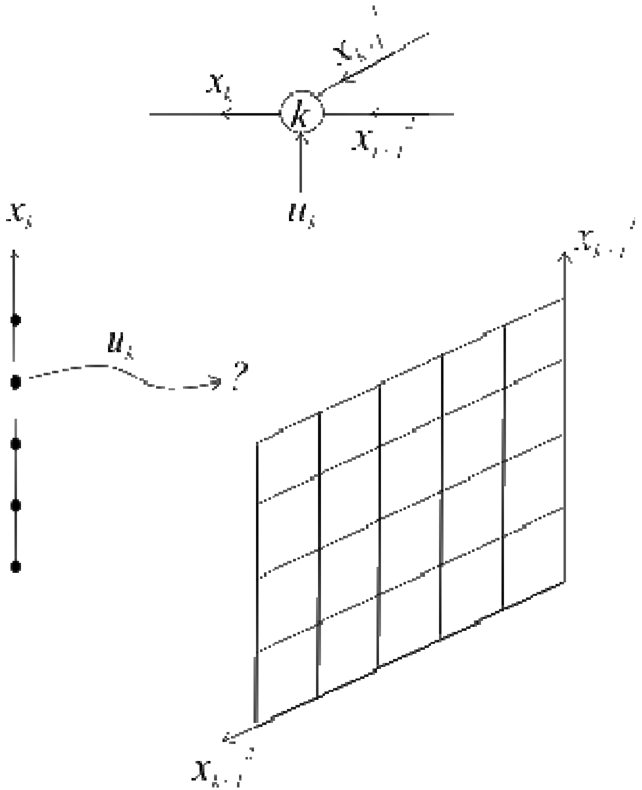




At node *k* we have a problem!

How to compute the contributions of stages *k+1'* and *k+1''*?

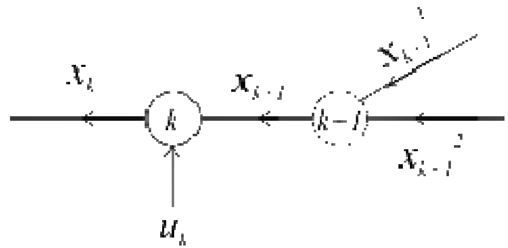
The New DP Approach

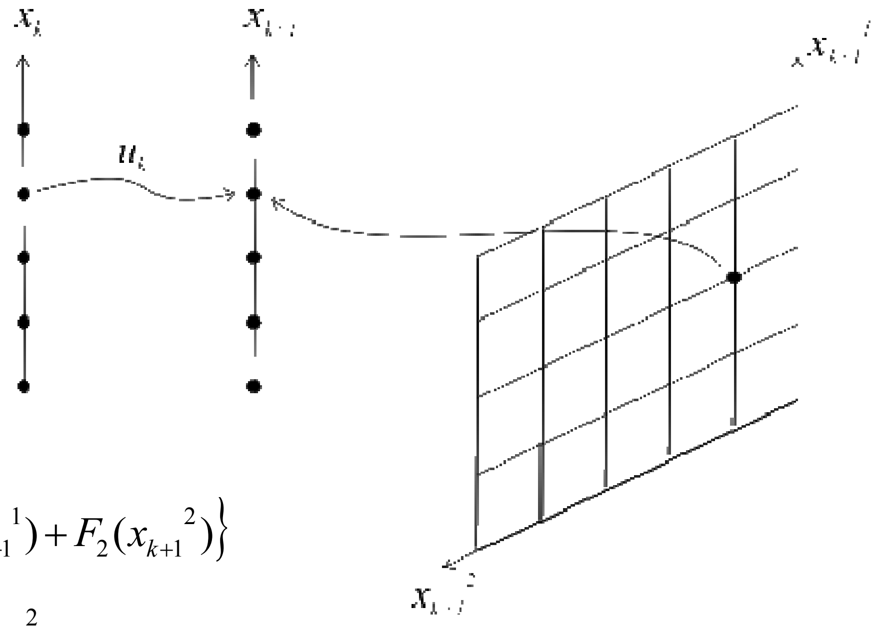


Does it need a multidimensional DP algorithm?

The New DP Approach

The capacitor allocation problem for networks with lateral branches is a “*false*” multidimensional DP problem.



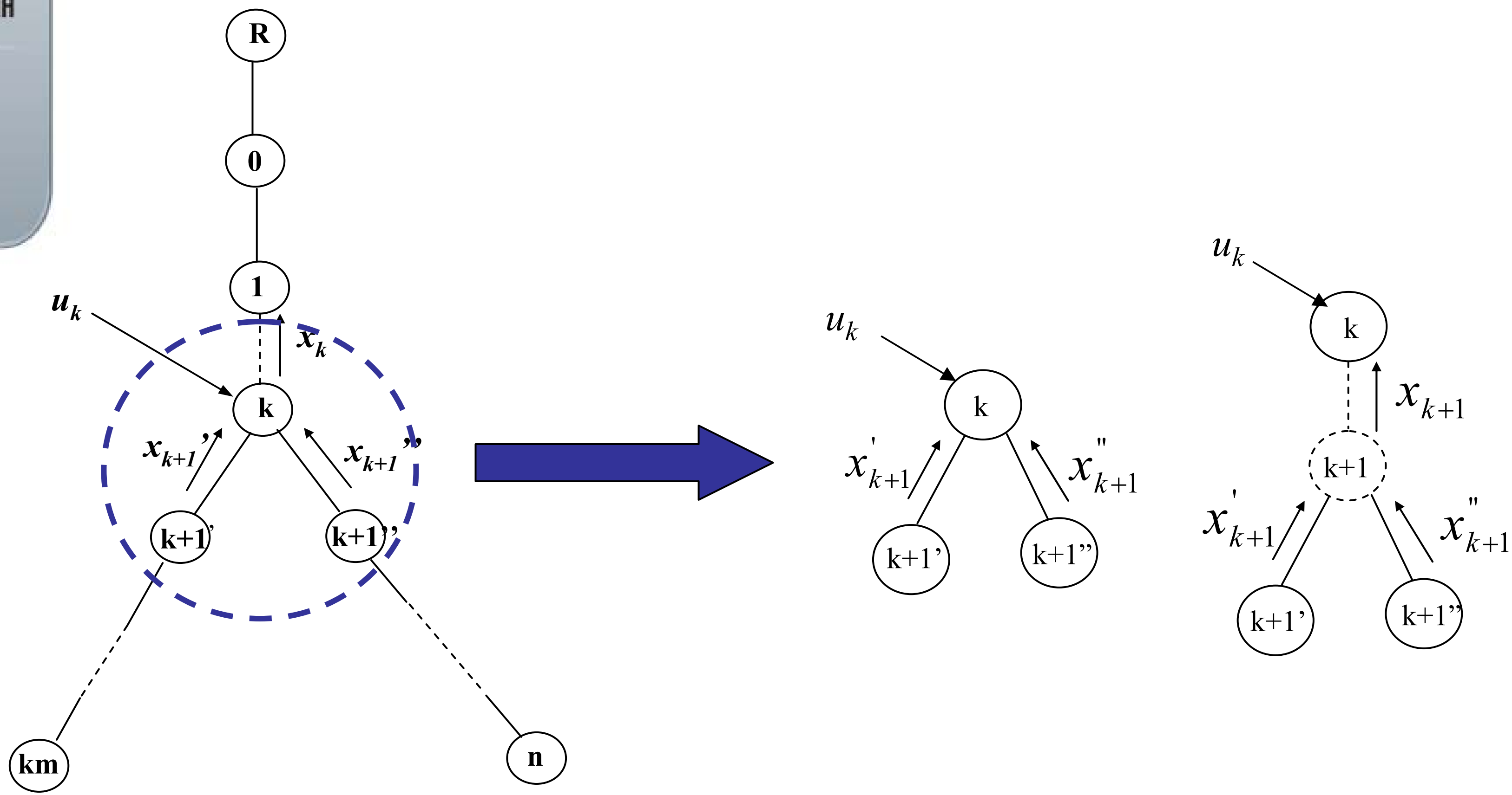


$$F(x_{k+1}) = min_{x_{k+1}^{1}, x_{k+1}^{2}} \left\{ F_1(x_{k+1}^{1}) + F_2(x_{k+1}^{2}) \right\}$$

$$x_{k+1} = x_{k+1}^{1} + x_{k+1}^{2}$$

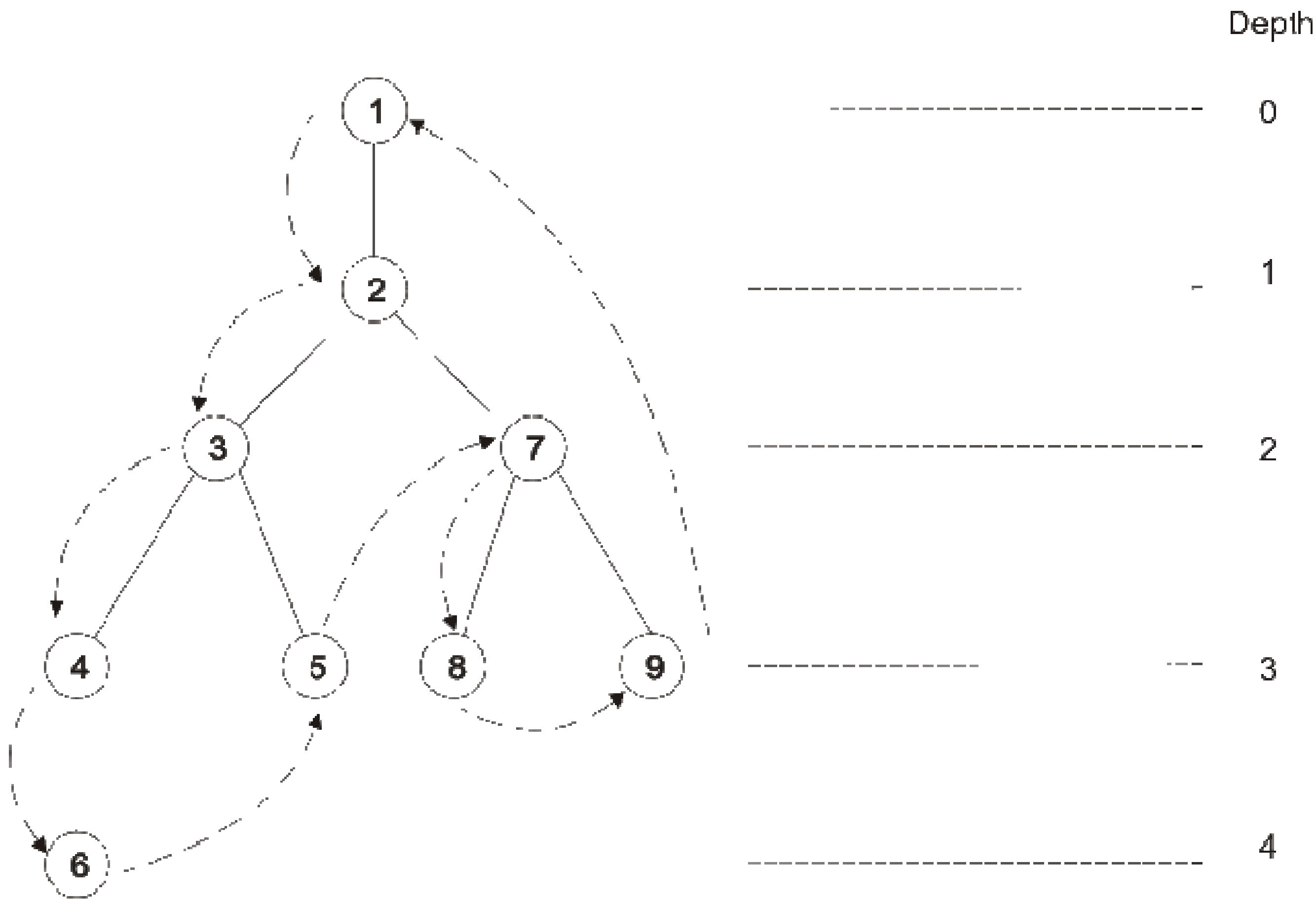
The New DP Approach

Projecting the problem into the virtual stage *k+1* avoids the need of more dimensions in the DP approach



How to solve it?

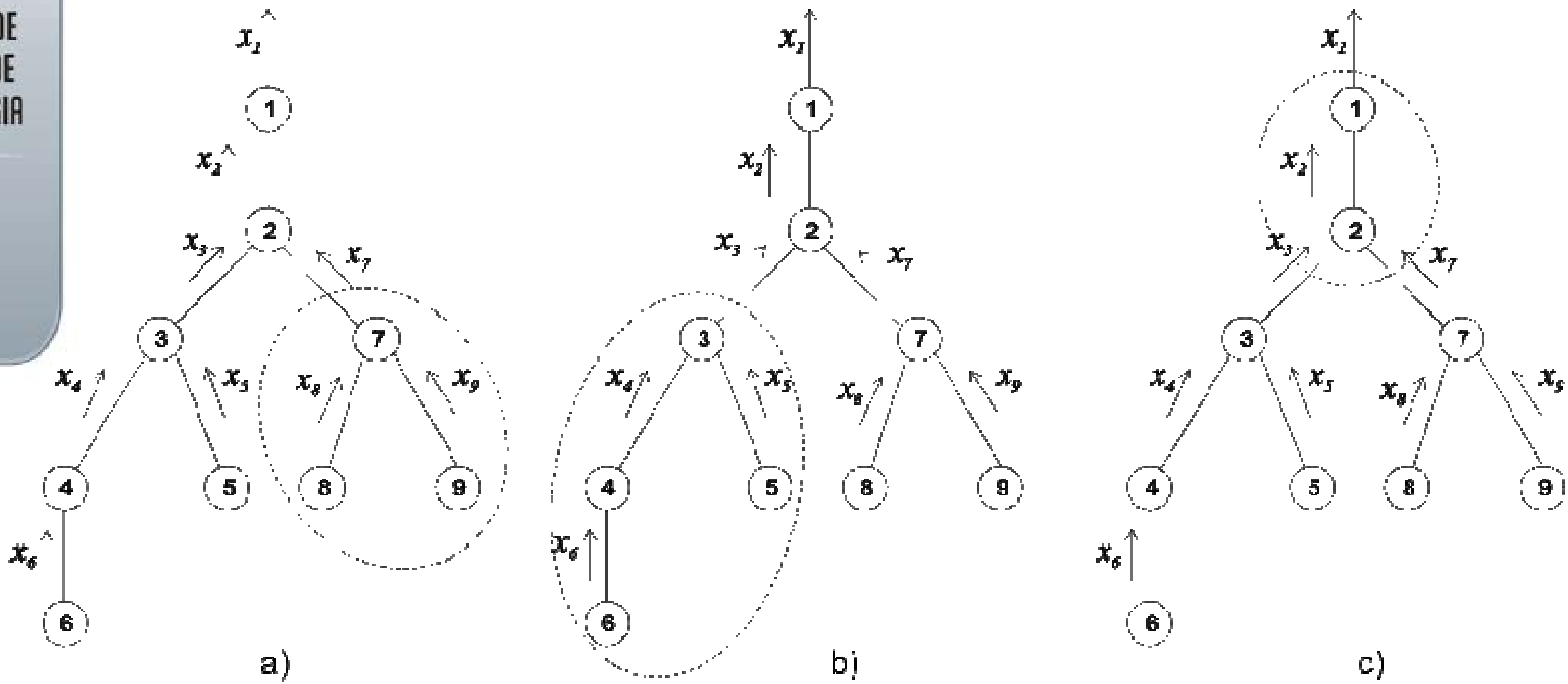
Borrowing ideas from NF algorithms.



The backward DP procedure traverses the network with paths inverse to preorder. In this example: 9-8-7-5-6-4-3-2-1

How to solve it?

DP applied to the example



a) Compute $F_9(x_9)$, $F_8(x_8)$ e $F_7(x_7)$;

b) Compute $F_5(x_5)$, $F_6(x_6)$, $F_4(x_4)$ *and* $F_3(x_3)$;

c) Compute $F_2(x_2)$ e $F_1(x_1)$;

d) Go forward (in preorder) finding the optimal solution.

A Flavor of Applications

The algorithms was coded in C++ (Borland C++ 5.5) and ran under Windows 2000™ in a Pentium 4 2.2 GHz system.

Instances A and B, with 1596 and 2448 nodes, respectively.

Energy cost: α_{et}=*0,08 R$/kwh*

Capacitor cost: k_c=*5,00 R$/kVAr*

One year, with intervals: τ_0=1000, τ_1=6760 e τ_2=1000 hours

Capacitors banks used: 150, 300, 450, 600, 900 and 1200 kVAr

Results

Instance	Initial Cost (R$)	Solution Cost (R$)	Installed Capacity (kVAr)	Savings (%)
A	197.335	186.907	1800	5,28
B	451.092	386.008	5400	14,43

Computational times were 0,172s and 0,297s, for *A* for *B*.

Discussion

- DP can be used to solve the fixed capacitor allocation problem (under the usual assumption of V_k= 1 pu).
- Borrowed key ideas from NF problems.
- It can address real scale systems.
- DP gives a global optimal solution.
- With an additional dimension the approach can be generalized to the switched capacitor allocation problem.
- What to do if $V_k \neq 1$ pu?

Acknowledgments

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FAPESP

Grazie!

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