The Uncapacitated Swapping Problem

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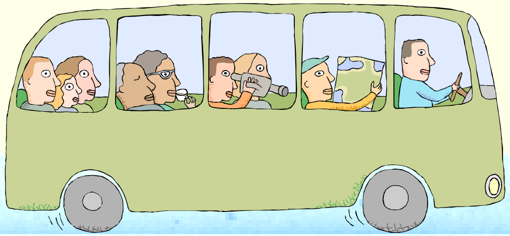
Outline

- Overview of relevant problems: the Swapping Problem, the DARP and the SCP
- Summary of literature results
- Results for the Uncapacitated SP:
  - $O(n \log n)$ exact – on a line
  - $O(n^3)$ exact – on a circle

Swapping Problem (SP)

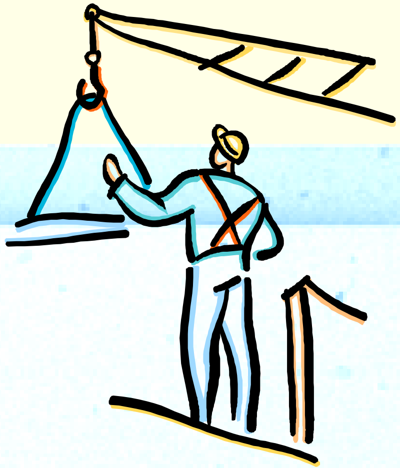
- A graph G(V,E), with n vertices
- m object types,
- Initial state: the object type at each vertex .
- Final state: the object type desired at each vertex.
- A single vehicle of given capacity.
- The SP is to compute a shortest route, along which the vehicle can accomplish the rearrangement of the objects.
- NP-hard

Dial-A-Ride Problem (DARP)



- The DARP can be seen as a particular case of the SP:
- Only one unit of each object type
- The single vehicle is of some finite capacity
- (Often there are additional practical constraints)

Stacker Crane Problem (SCP)



- The SCP can also be seen as a particular case of the SP:
- only one unit of each object type
- a unit capacity vehicle
- (all the objects are non-preemptive)

Possible Variations

Graph Structure	Line, Circle, Tree, General Graph
Vehicle's Capacity, **k**	k=1, k>1, infinite k
# of Object Types, **m**	m=1, m=2, m>2, null object
Objects' Nature	Preemptive, Non-preemptive, Mixed
Stations' Capacity	One, Greater than one
Vehicle's Load	Mixture of different object types in the vehicle is possible or not

Summary of Results (1)

Results for the Unit Capacity Case (k=1)

	Swapping Problem			Stacker Crane Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	Non-preemptive	Mixed
Line	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial
Circle	★	★	★	Polynomial	Polynomial	Polynomial
Tree	NP-hard	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

From: Anily, Gendreau and Laporte, 2006

Summary of Results (2)

Results for the Finite Capacity Case (k>1)

	Swapping Problem			Dial-a-Ride Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	Non-preemptive	Mixed
Line	★	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
Circle	★	NP-hard	NP-hard	★	NP-hard	NP-hard
Tree	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Summary of Results (3)

Results for the Infinite Capacity Case (k=∞)

	Swapping Problem	Dial-a-Ride Problem
Graph Structure	Non-preemptive	Non-preemptive
Line	Polynomial	Polynomial
Circle	Polynomial	Polynomial
Tree	★	★
General	NP-hard	NP-hard

Uncapacitated SP on a Line

Notations

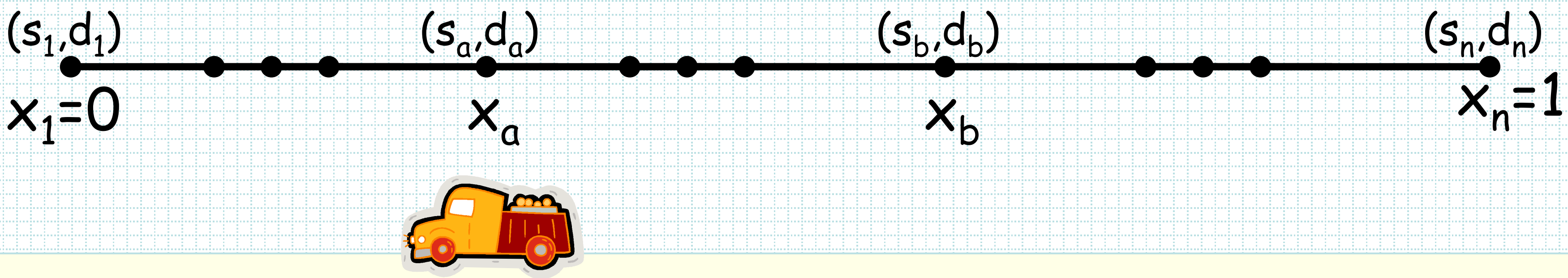
n stations, i=1,…,n

Xi = distance from left endpoint

Xa = starting location; Xb = ending location

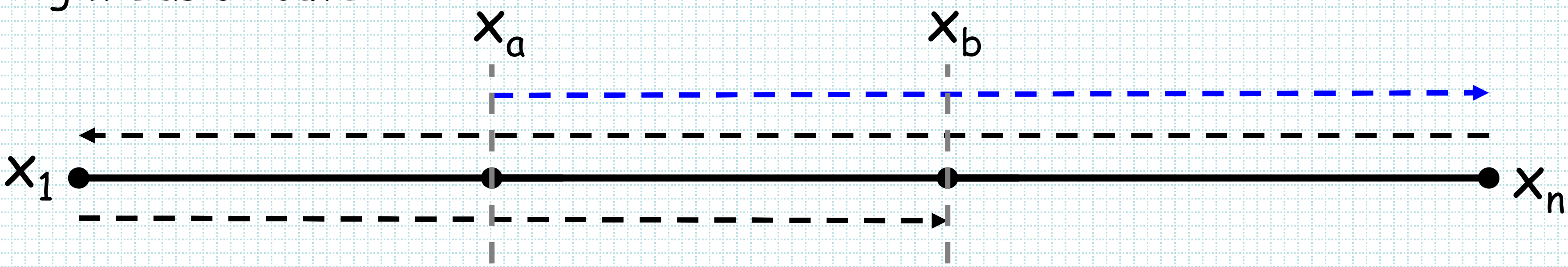
m object types, j=1,…,m

Initial arrangement - si; final arrangement - di

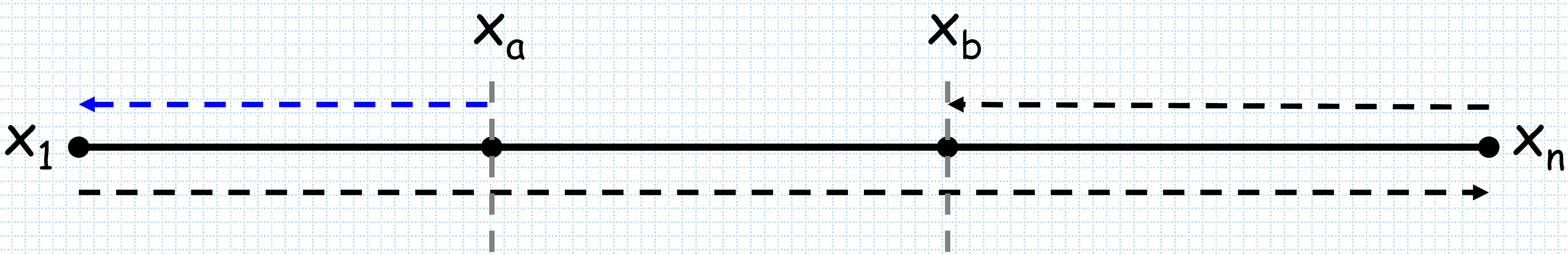


Two Basic Routes

Right Basic Route:



Left Basic Route:

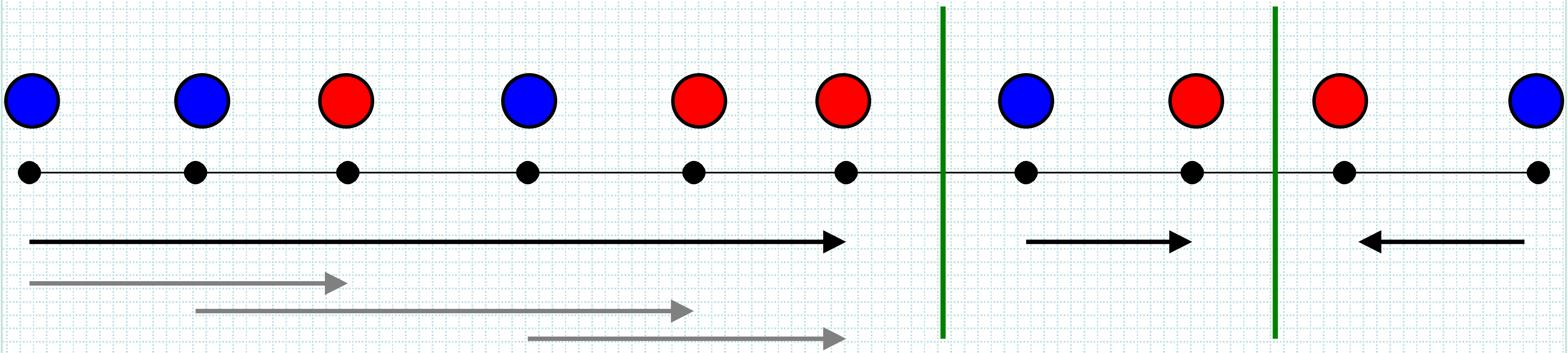


Consecutive Minimally Balanced Partition

● Supply

● Demand

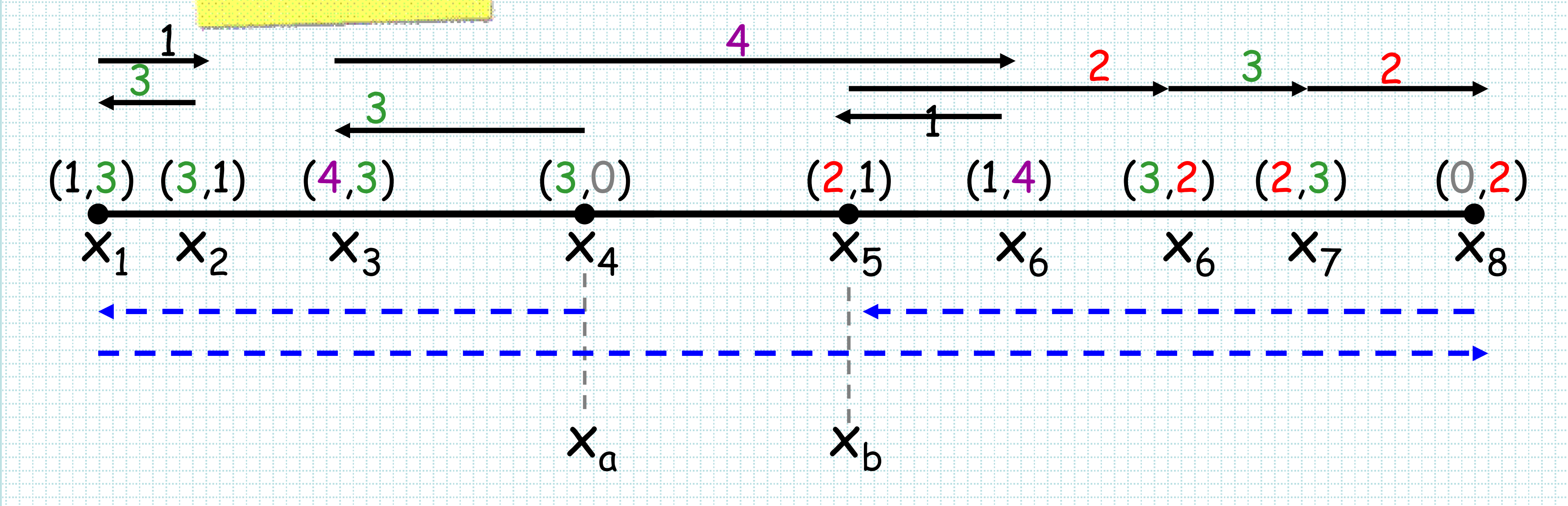
<u>Property</u>: In every optimal solution the vehicle travels **loaded** (at least) along all the (intervals covered by) minimally balanced arrows.



Tight Lower & Upper Bounds

Lower Bound

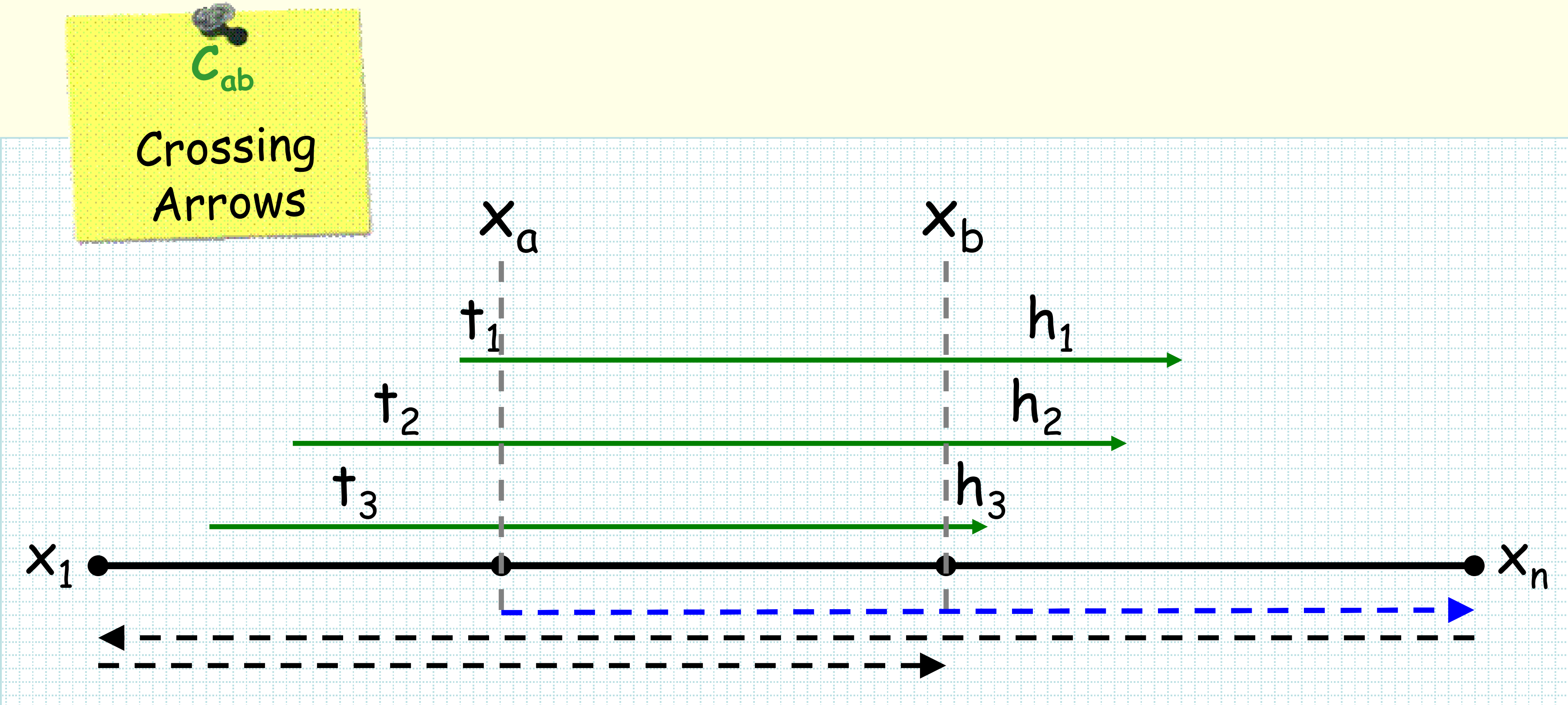
$$LB = 1 + x_a + (1 - x_b) = 2 + x_a - x_b$$



Upper Bound

$$UB = Min\{2 + x_a + x_b, 4 - (x_a + x_b)\}$$

Using the Right Basic Route



$$H_i = \max_{j>1} h_j = h_{i+1}$$

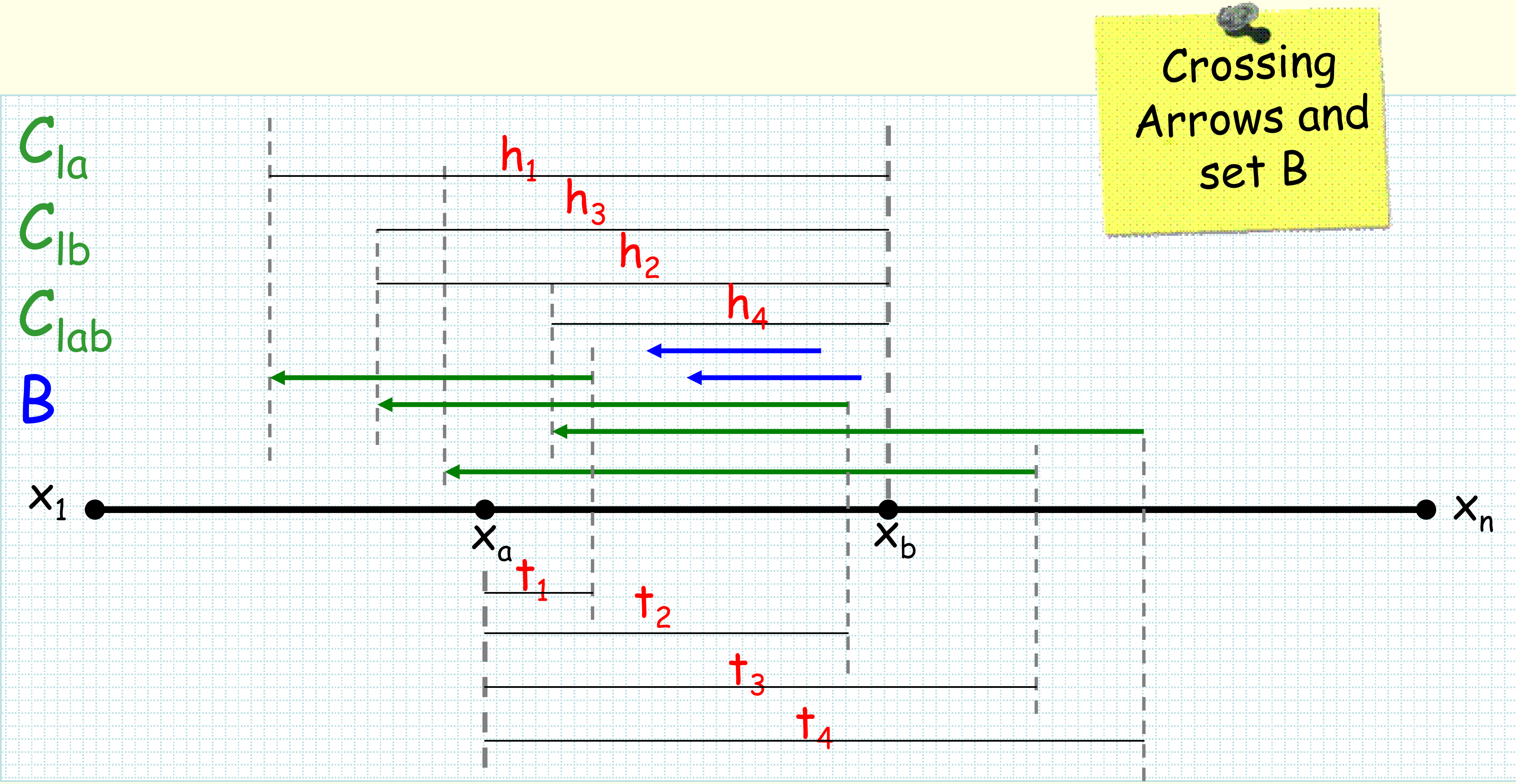
Using the Right Basic Route

- Sort all crossing arrows in C_{ab} in an ascending order of their tail - part
- Eliminate all covered crossing arrows
- Compute $L_{right} = \underset{i}{Min}\{t_i + h_{i+1}\}$

$$F_{right} = (2 + x_b - x_a) + 2L_{right}$$

$O(n \log n)$

Using the Left Basic Route



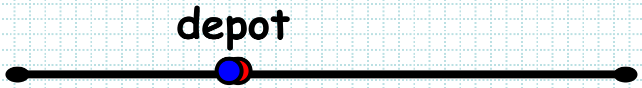
Using the Left Basic Route

- Sequences in B not (partially) covered – in loops (**LB**)
- Sort all crossing arrows in an ascending order of their tail - part
- Eliminate all covered crossing arrows
- Compute $L_{right} = LB + \underset{i}{Min}\{t_i + h_{i+1}\}$

$F_{left} = (2 + x_a - x_b) + 2L_{left}$

$O(n \log n)$

Two Special Cases

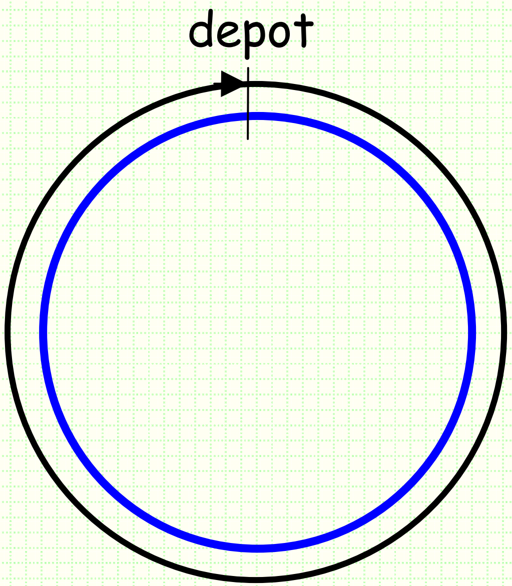
SP-Line(d)		O(nlogn)
SP-Line(0,1)		O(n)

Uncapacitated SP on a Circle

Bounds

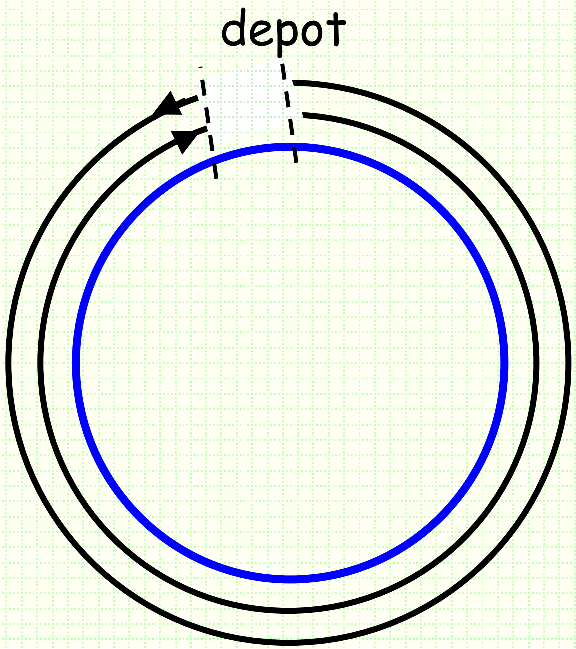
Lower Bound

1

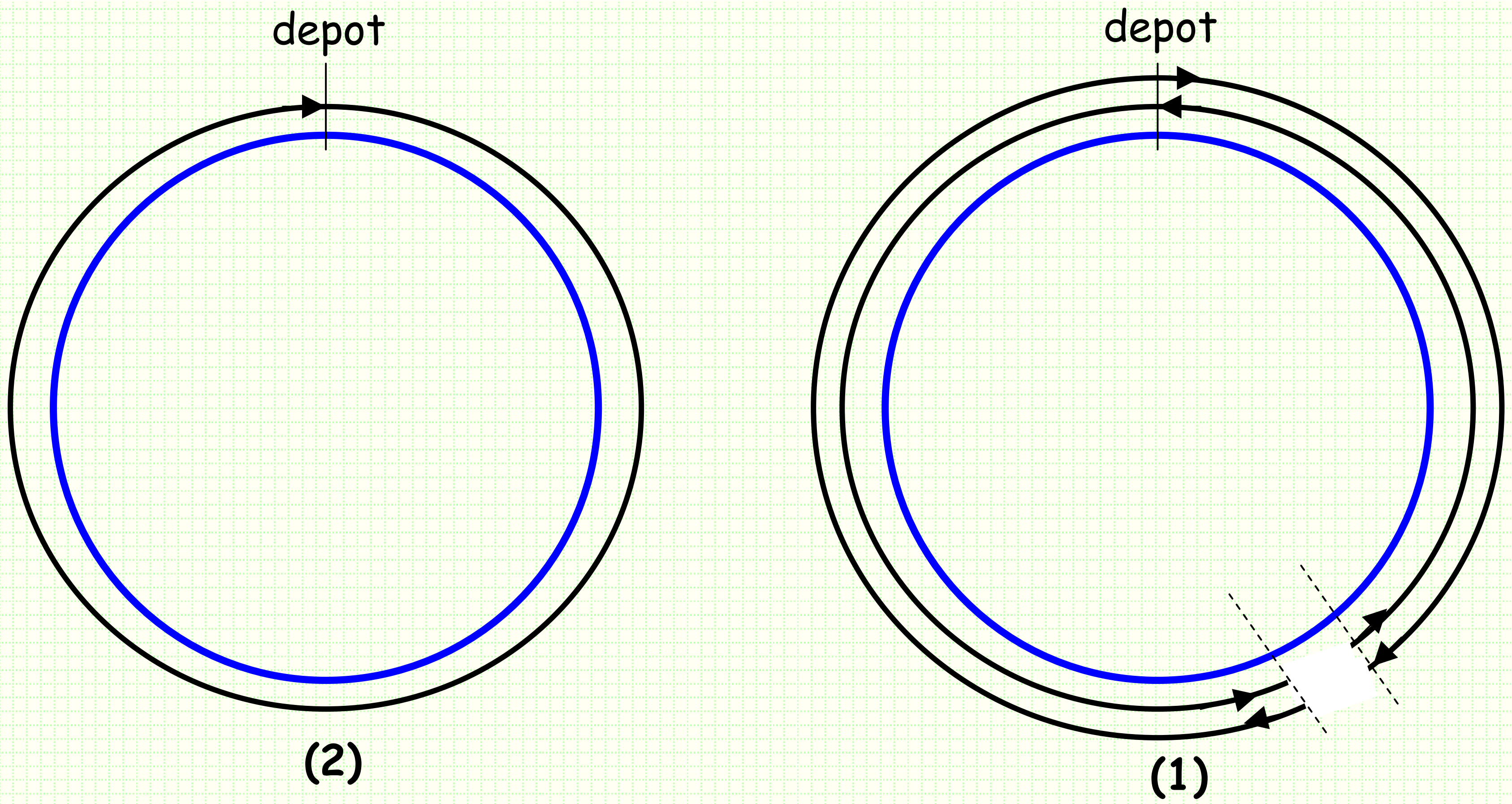


Upper Bound

$2(1 - \max\{l_1, l_n\})$



Uncapacitated SP on a Circle

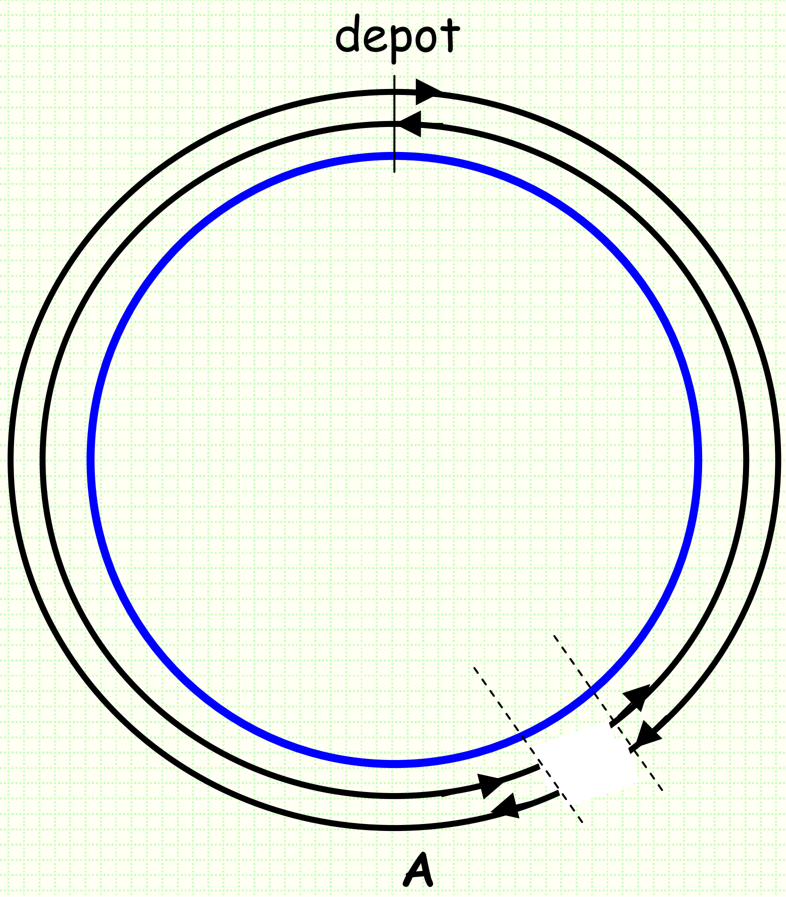


(1) One interval not covered

Case 1:

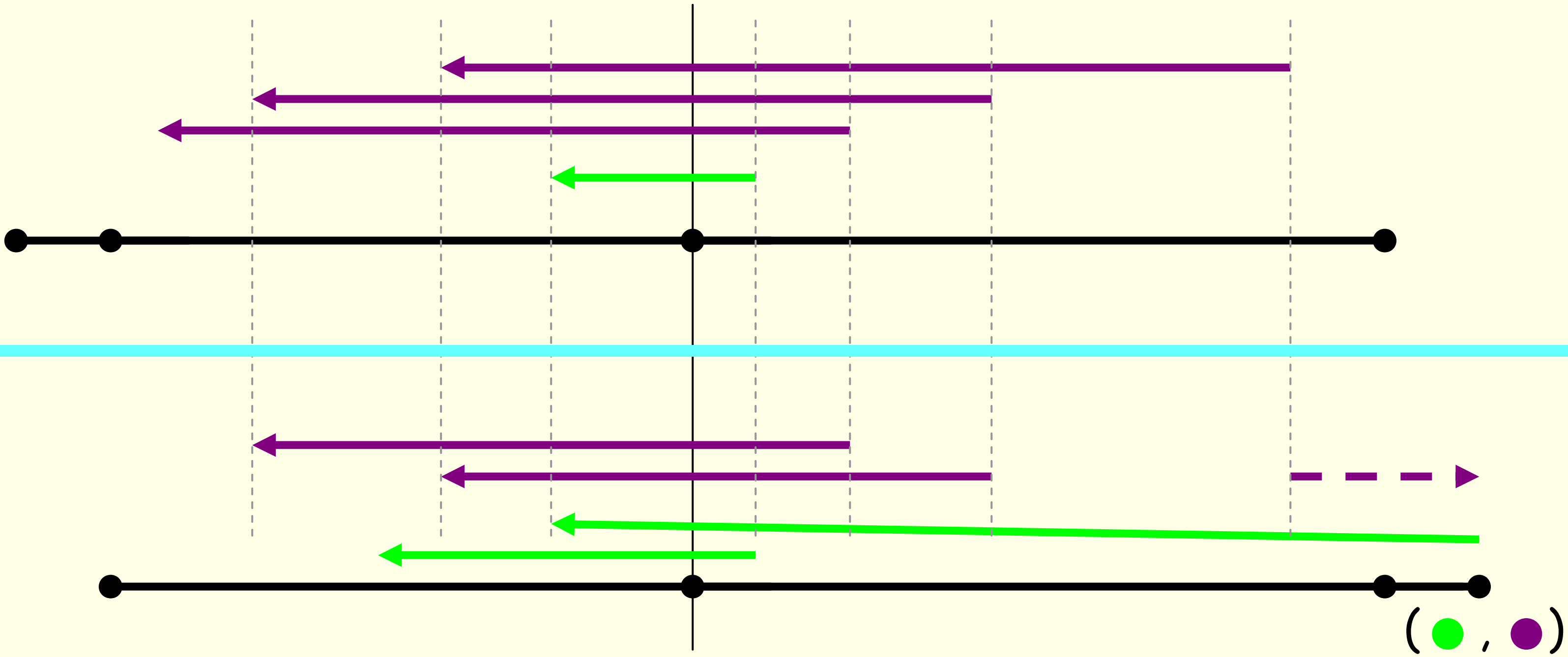
one interval not covered

- For each interval not covered: SP-Line(d)
- For both right and left basic routes:
- **Update** the lists ti, hi, **Hi**
- Compute ti+Hi



O(n^2)

(1) One interval not covered



t	h	t	h
3	**11**	1	2
6	8		
12	5		

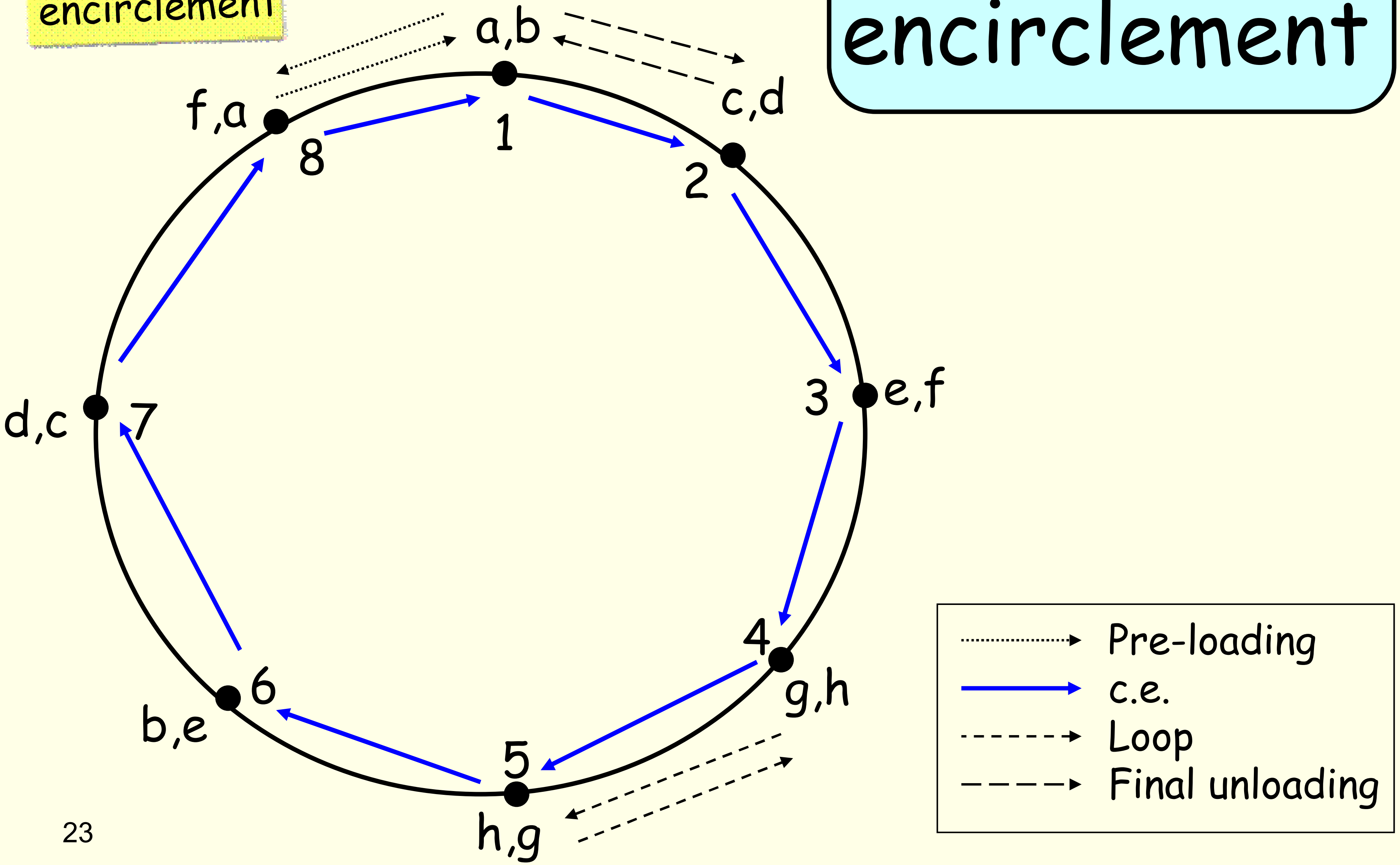
→

t	h	t	h
3	8	1	6
6	5	15	2

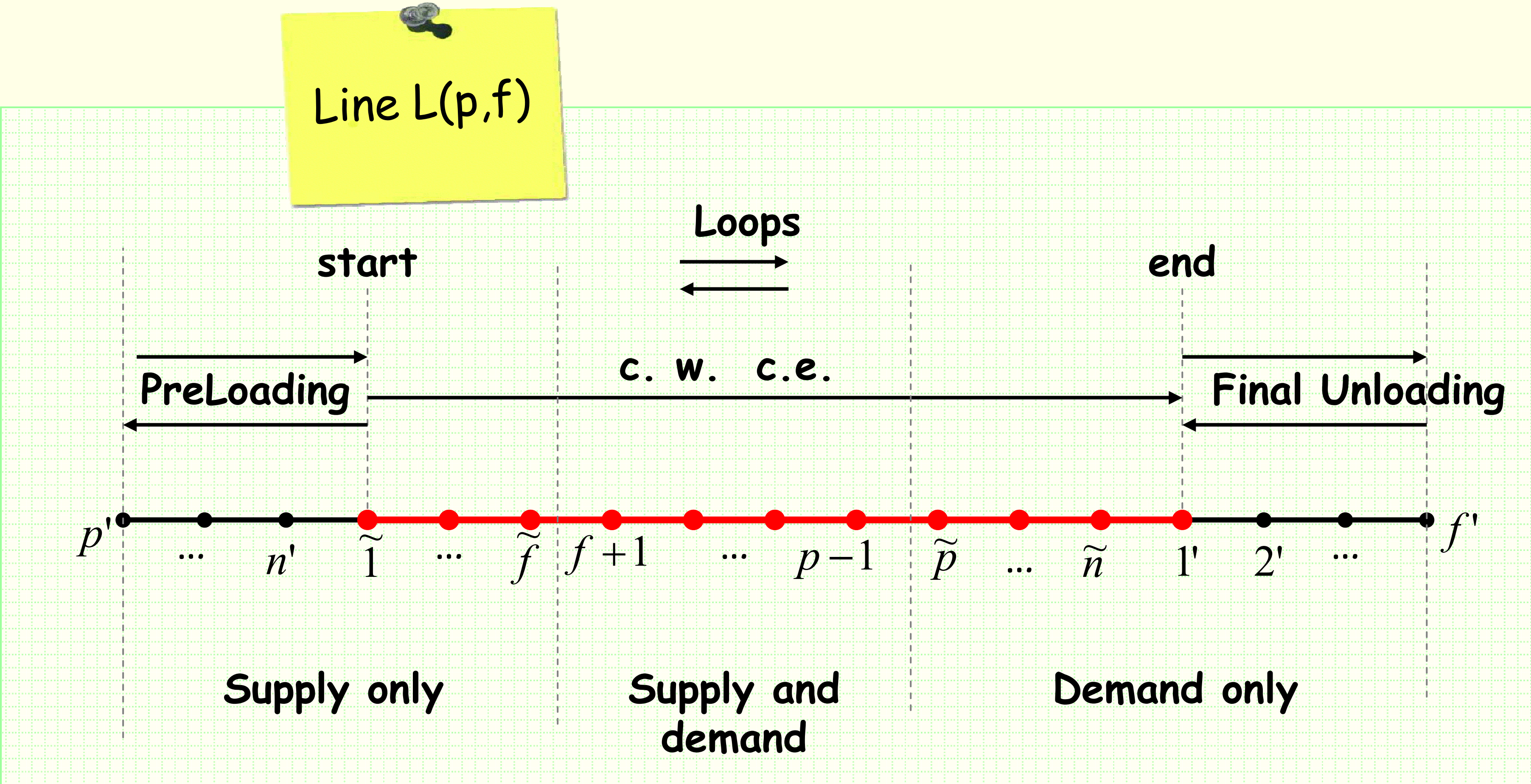
(2) Complete encirclement

Case 2:

Complete encirclement



The Line L(p,f)



Line L(p,f)
Loops
start
end
PreLoading
c. w. c.e.
Final Unloading
p'
…
n'
$\tilde{1}$
…
$\tilde{f}$
$f+1$
…
$p-1$
$\tilde{p}$
…
$\tilde{n}$
1'
2'
…
f'
Supply only
Supply and demand
Demand only

(2) Complete encirclement

Claim: Solving the SP on the line L(p,f) with starting point in station $\tilde{1}$ and ending point in station 1' is **equivalent** to solving the SP on a circle with starting and ending at the depot at station 1, when the preloading contains the stations from 1 to p c.c.w., and the final-unloading contains the stations from 1 to f c.w.

- For every relevant pair of p and f:
- Solve the problem on the line L(p,f)

$O(n^3)$

Thank You!

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Summary of Results (1)

Results for the Unit Capacity Case (k=1)

Anily, Gendreau and Laporte (1999).
$O(n^2)$ exact algorithm

	Swapping Problem			Stacker Crane Problem		
Graph Structure	Preemptive	pre			ve	Mixed
Line	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial
Circle	★	★	★	Polynomial	Polynomial	Polynomial
Tree	NP-hard	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

From: Anily, Gendreau and Laporte, 2006

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Graph Structure	Preemptive	Non-preemptive	Mixed	Pree		ixed
Line	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial
Circle	★	★	★	Polynomial	Polynomial	Polynomial
Tree	NP-hard	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Atallah and Kosaraju (1988).
O(m+n) exact algorithm

From: Anily, Gendreau and Laporte, 2006

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	Swapping Problem			Stacker Crane Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptiv		
Line	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial
Circle	★	★	★	Polynomial	Polynomial	Polynomial
Tree	NP-hard	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Atallah and Kosaraju (1988). O(m+nα(n)) exact algorithm

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Graph Structure	Preemptive	Non-preemptive	Mixed			Mixed
Line	Polynomial	Polynomial	Polynomial			Polynomial
Circle	★	★	★	Polynomial	Polynomial	Polynomial
Tree	NP-hard	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Atallah and Kosaraju (1988).
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Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	N[illegible]	[illegible]
Line	Polynomial	Polynomial	Polynomial	Polynomia[illegible]	[illegible]	[illegible]l
Circle	★	★	★	Polynomial	Polynomial	Polynomial
Tree	NP-hard	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Atallah and Kosaraju (1988).
O(m+nlogn) exact algorithm

From: Anily, Gendreau and Laporte, 2006

Summary of Results (1)

Results for the Unit Capacity Case (k=1)

	Swapping Problem			Stacker Crane Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	Non-preemptive	Mixed
Line	Polynomial	Polynomial	Po			nomial
Circle	★	★				ynomial
Tree	NP-hard	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Frederickson and Guan (1992).
O(m+nq) exact algorithm

From: Anily, Gendreau and Laporte, 2006

Summary of Results (1)

Results for the Unit Capacity Case (k=1)

	Swapping Problem			Stacker Crane Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	Non-preemptive	Mixed
Line	Polynomial	Polynomial	Polyno			l
Circle	★	★	★			al
Tree	NP-hard	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Frederickson and Guan (1993).
α=1.25 approximation algorithm

From: Anily, Gendreau and Laporte, 2006

Summary of Results (1)

Results for the Unit Capacity Case (k=1)

	Swapping Problem			Stacker Crane Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	Non-preemptive	Mixed
Line			ial	Polynomial	Polynomial	Polynomial
Circle				Polynomial	Polynomial	Polynomial
Tree	NP-hard	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Anily, Gendreau and Laporte (2006).
α=1. 5 approximation algorithm

From: Anily, Gendreau and Laporte, 2006

Summary of Results (1)

Results for the Unit Capacity Case (k=1)

Graph Structure	Swapping Problem: Preemptive	Swapping Problem: Non-preemptive	Swapping Problem: Mixed	Stacker Crane Problem: Preemptive	Stacker Crane Problem: Non-preemptive	Stacker Crane Problem: Mixed
Line	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial
Circle	★	★	[illegible]	[illegible]	Polynomial	Polynomial
Tree	NP-hard	NP-h[illegible]	[illegible]	[illegible]	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Anily and Hassin (1992).
α=2.5 approximation
algorithm

From: Anily, Gendreau and Laporte, 2006

Summary of Results (1)

Results for the Unit Capacity Case (k=1)

	Swapping Problem			Stacker Crane Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	Non-preemptive	Mixed
Line	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial
Circle	★	★	★	[illegible]	[illegible]	al
Tree	NP-hard	NP-hard	NP-har	[illegible]	[illegible]	d
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Frederickson, Hecht and Kim (1978).
α=9/5 approximation algorithm

From: Anily, Gendreau and Laporte, 2006

Summary of Results (2)

Results for the Finite Capacity Case (k>1)

	Swapping Problem			Dial-a-Ride Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed			
Line	★	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
Circle	★	NP-hard	NP-hard	★	NP-hard	NP-hard
Tree	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Guan (1998).
Charikar and Raghavachari (1998) α=2 approximation algorithm

Summary of Results (2)

Results for the Finite Capacity Case (k>1)

	Swapping Problem			Dial-a-Ride Problem		
Graph Structure	Preemptive	Non-preemptive	Mi		on- mptive	Mixed
Line	★	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
Circle	★	NP-hard	NP-hard	★	NP-hard	NP-hard
Tree	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Guan (1998). O(m+n) exact algorithm

Summary of Results (2)

Results for the Finite Capacity Case (k>1)

	Swapping Problem			Dial-a-Ride Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	Non-preemptive	Mixed
Line	★	NP-hard			NP-hard	NP-hard
Circle	★	NP-hard			NP-hard	NP-hard
Tree	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

> Charikar and Raghavachari (1998). α=2 approximation algorithm

Summary of Results (2)

Results for the Finite Capacity Case (k>1)

	Swapping Problem			Dial-a-Ride Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	Non-preemptive	Mixed
Line	★	NP-hard	NP-hard	Poly		
Circle	★	NP-hard	NP-hard			
Tree	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Charikar and Raghavachari (1998). $\alpha=O(\sqrt{k})$ approximation algorithm

Summary of Results (2)

Results for the Finite Capacity Case (k>1)

	Swapping Problem			Dial-a-Ride Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	Non-preemptive	Mixed
Line	★	NP-hard	N[illegible]	[illegible]	[illegible]rd	NP-hard
Circle	★	NP-hard	[illegible]	[illegible]	[illegible]d	NP-hard
Tree	NP-hard	NP-hard	NP-ha[illegible]	[illegible]-hard	N[illegible]-hard	NP-hard
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Charikar and Raghavachari (1998). $\alpha = O(\log n \log\log n)$ approximation algorithm

Summary of Results (2)

Results for the Finite Capacity Case (k>1)

	Swapping Problem			Dial-a-Ride Problem		
Graph Structure	Preemptive	Non-preemptive	Mixed	Preemptive	Non-preemptive	Mixed
Line	★	NP-hard	NP-hard	Polynomial	NP-hard	NP-hard
Circle	★	NP-hard	NP-hard	★		
Tree	NP-hard	NP-hard	NP-hard	NP-		
General	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard	NP-hard

Charikar and Raghavachari (1998).
$\alpha=O(\sqrt{k}\log n\log\log n)$
approximation algorithm

Summary of Results (3)

Results for the Infinite Capacity Case (k=∞)

This work

	Swapping Problem	Dial-a-Ride Problem
Graph Structure	tive	Non-preemptive
Line	Polynomial	Polynomial
Circle	Polynomial	Polynomial
Tree	★	★
General	NP-hard	NP-hard

Summary of Results (3)

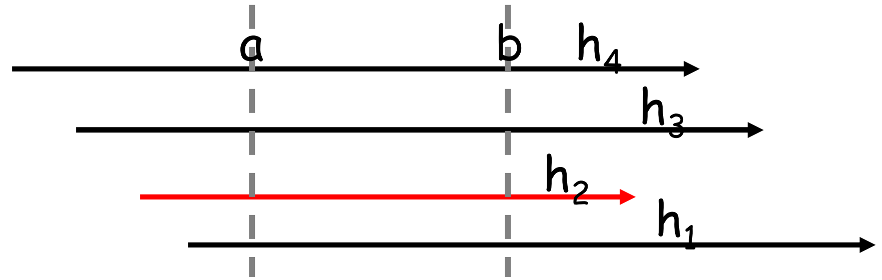
Results for the Infinite Capacity Case (k=∞)

Graph Structure	Swapping Problem	Dial-a-Ride Problem
Graph Structure	Non-preemptive	Non-preemptive
Line	Polynomial	Polynomial
Circle	Polynomial	
Tree	★	
General	NP-hard	NP-hard

Kubo and Kasugai (1999).
Psaraftis (1983).
α=3 approximation

Usin…

- Let h:=hc
- **For** i← c-1 downto 1 do
  - **If** hi<=h **then** delete arrow i,
  - **Else** update h=hi



X

- Sort c… ascen…
- **Eliminate all covered** crossing arrows
- Compute $L_{right} = \underset{i}{Min}\{t_i + h_{i+1}\}$

$$F_{right} = (2 + x_b - x_a) + 2L_{right}$$

$O(n\log n)$