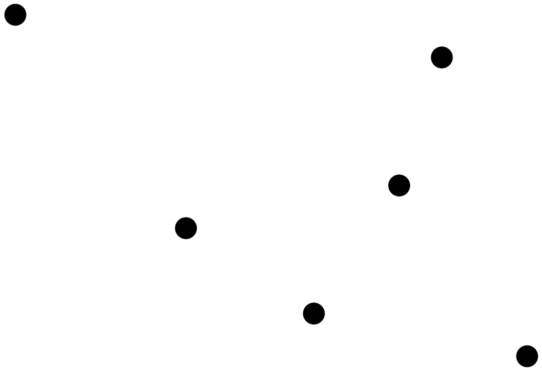
A Simple 3-Approximation of Minimum Manhattan Networks

Bernhard Fuchs and Anna Schulze

TU Braunschweig and Uni Köln

CTW 2008

Manhattan networks



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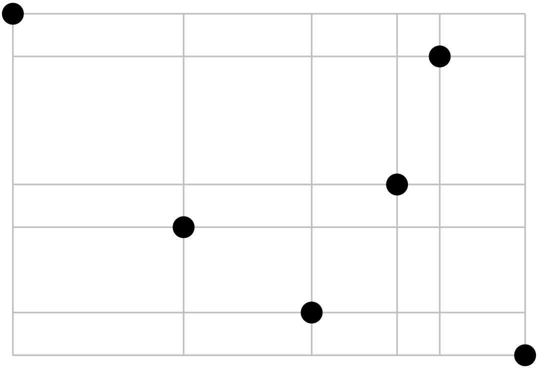

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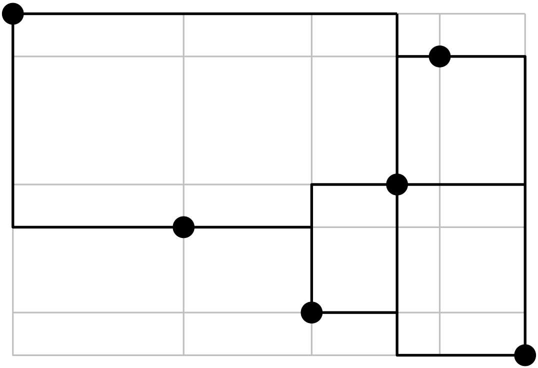

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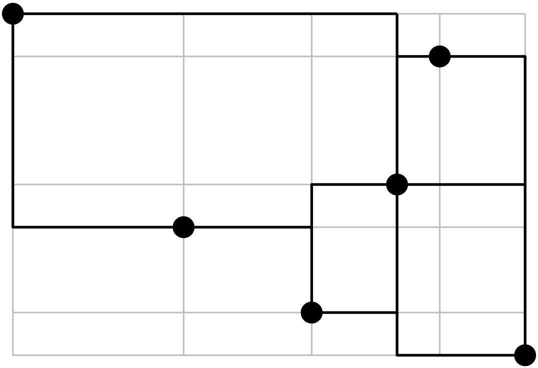

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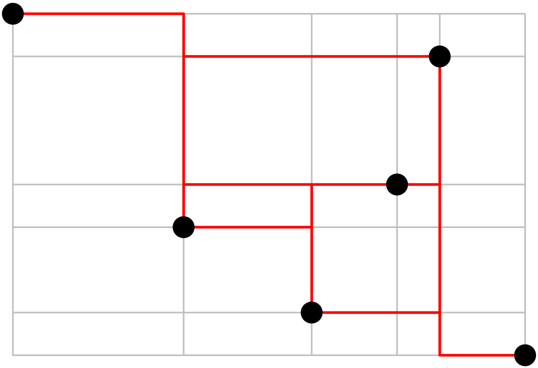

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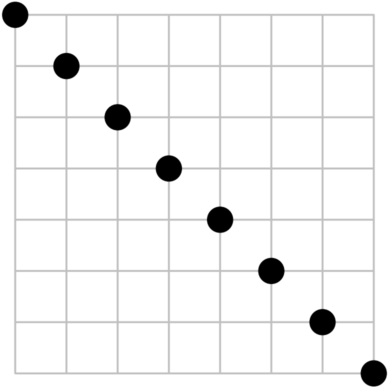
Simple heuristics – Hannan grid



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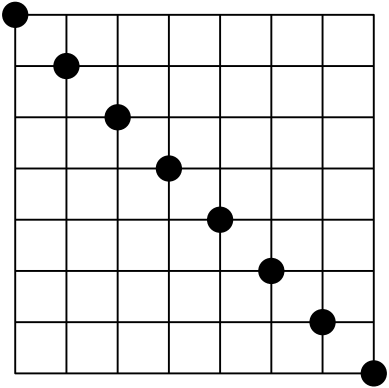

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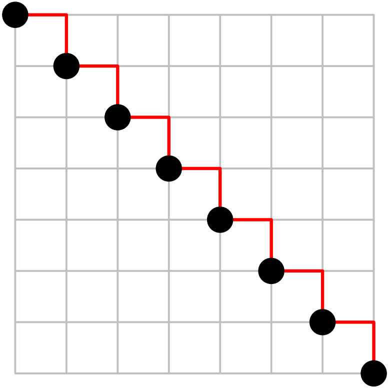

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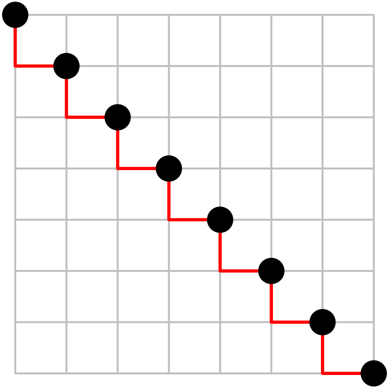

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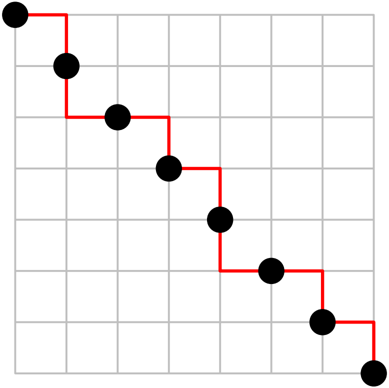

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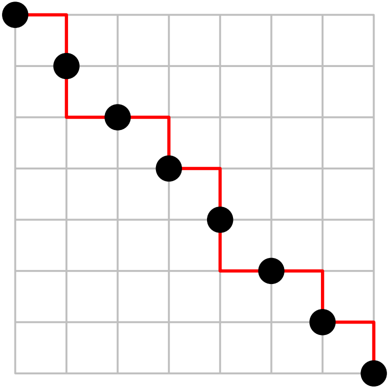

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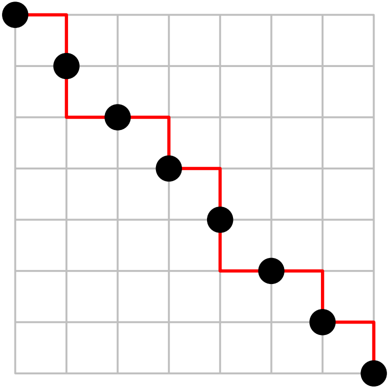

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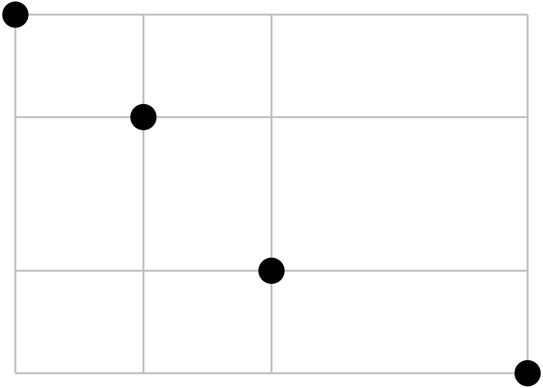
- Next idea: Just insert **critical rectangles**!

Critical rectangles



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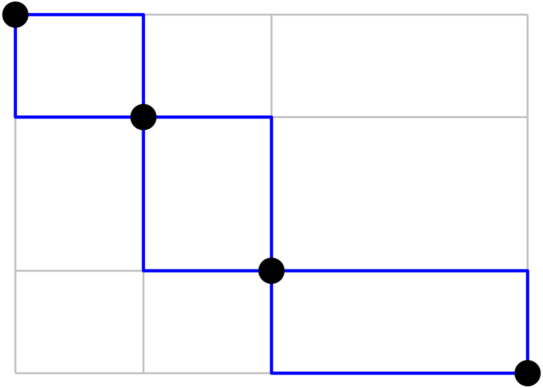

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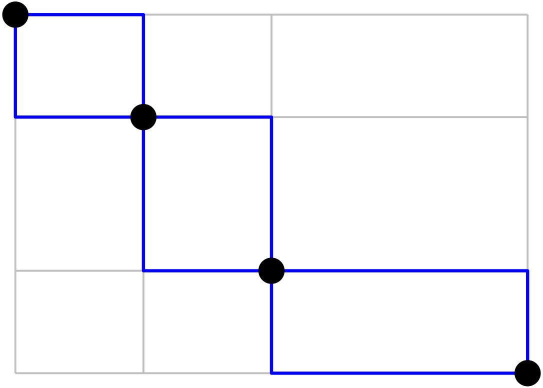

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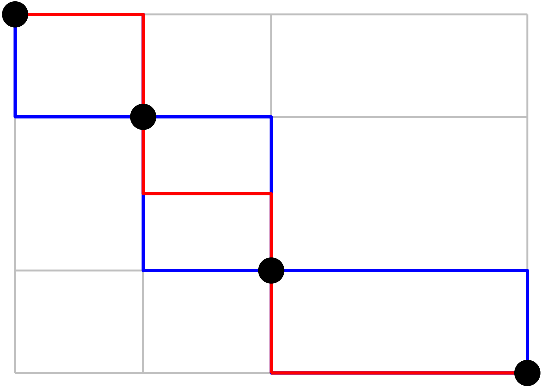

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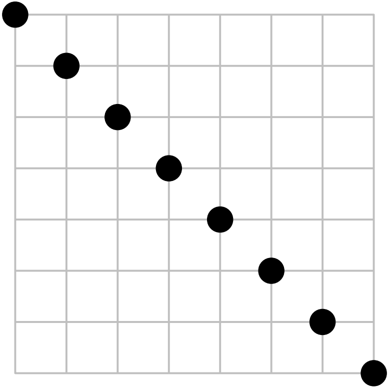

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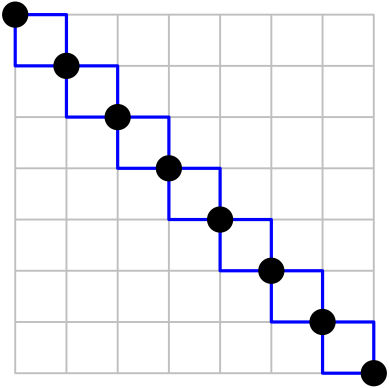

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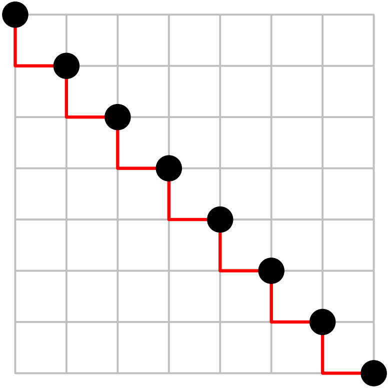

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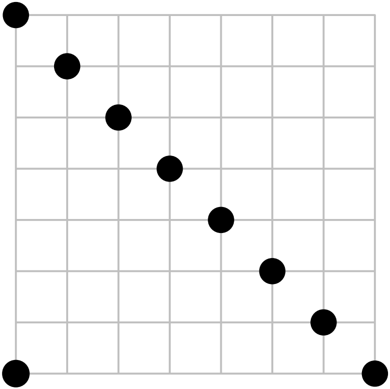

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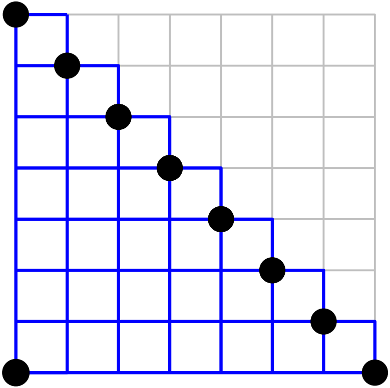

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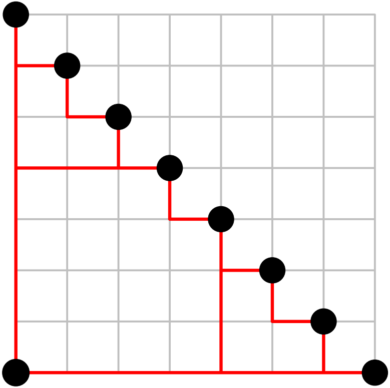

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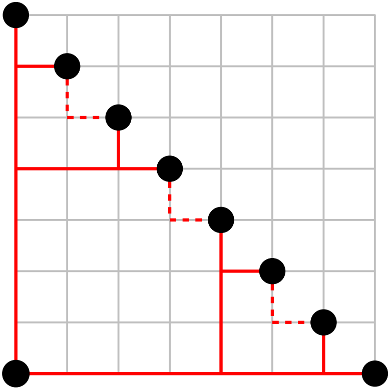

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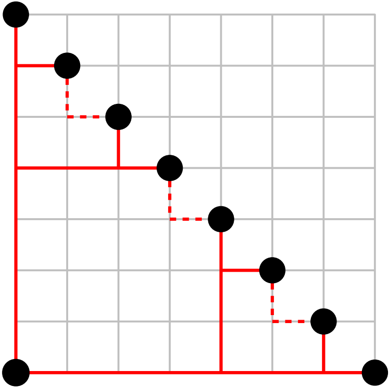

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- Much simpler than [Benkert et al.], both algorithm and proof.

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Phase II

A standard 2-approximation algorithm inside so-called '*staircases*'.

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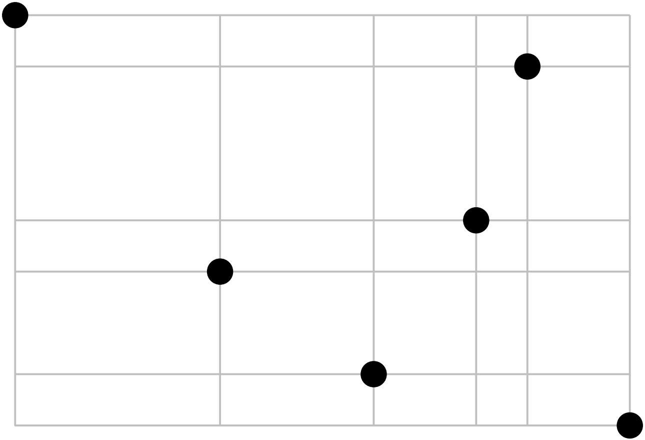

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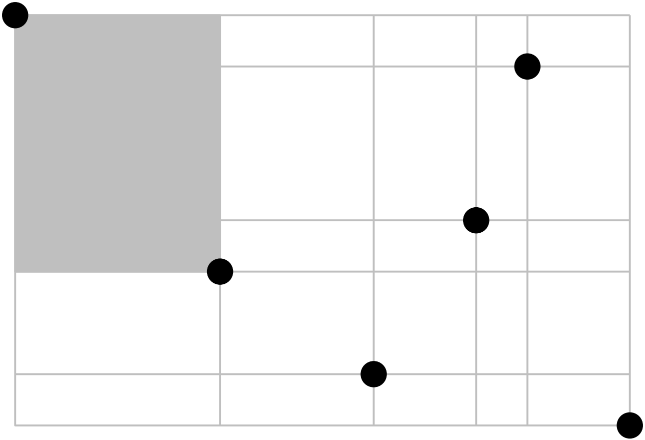

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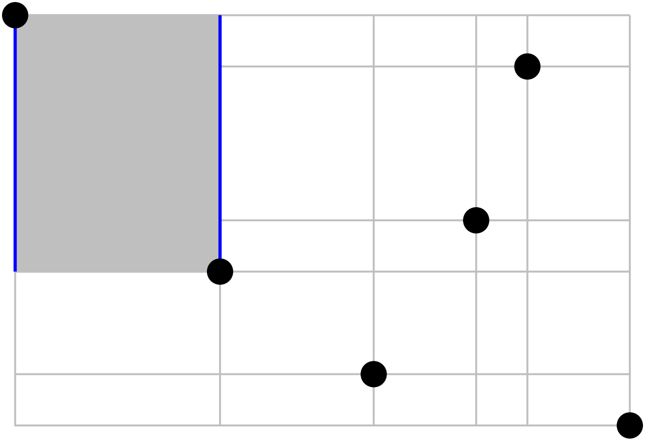

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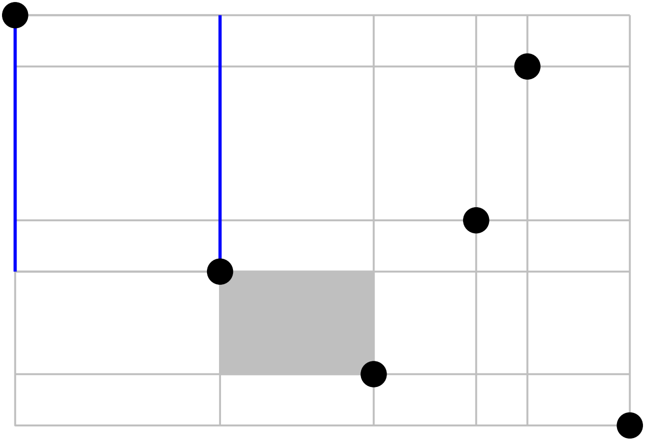

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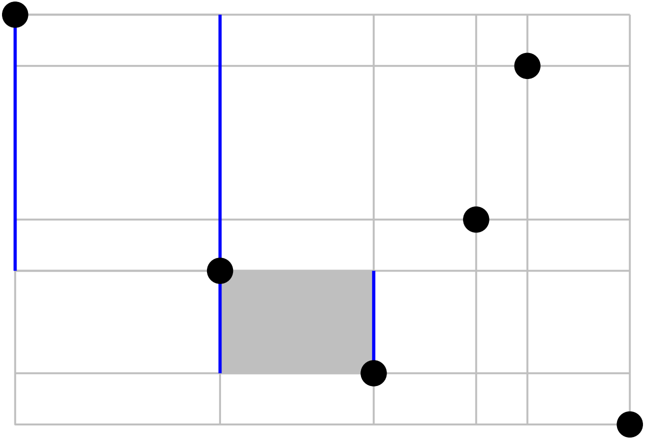

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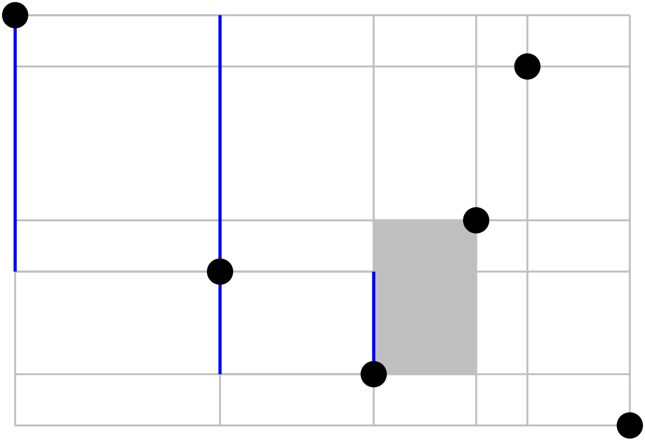

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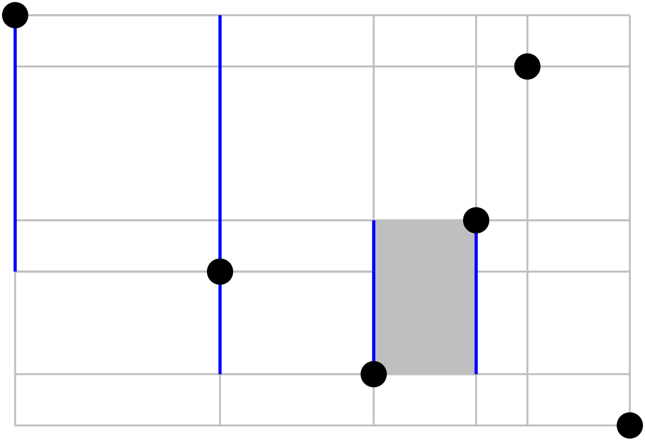

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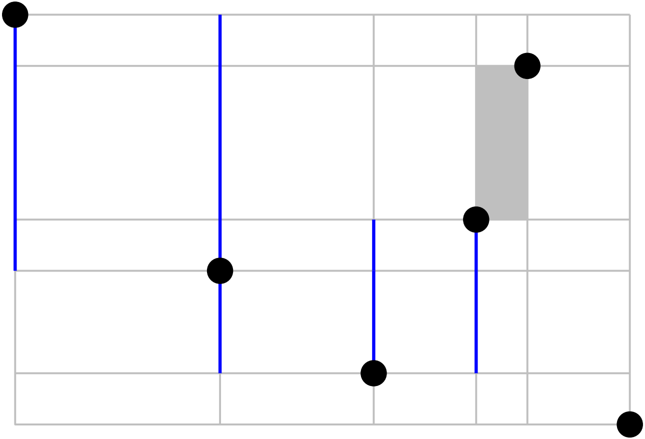

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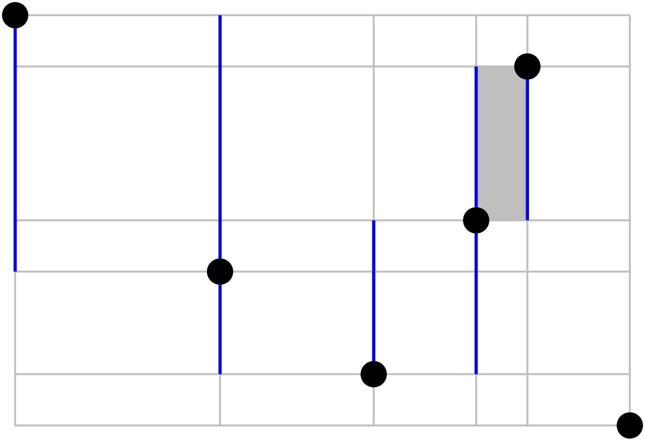

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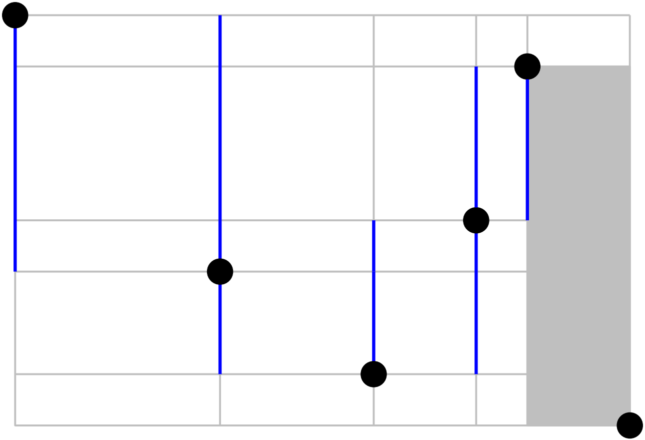

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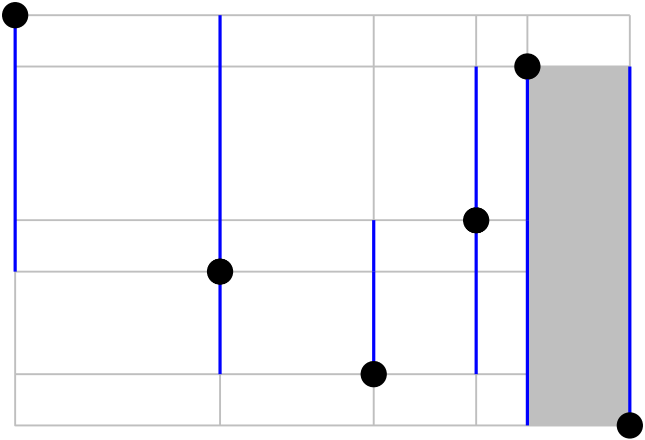

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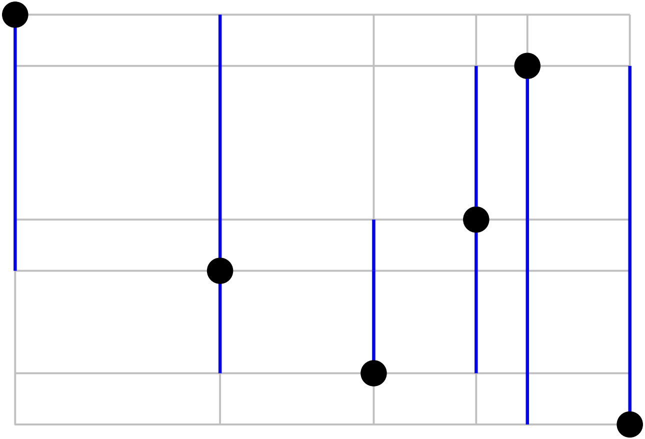

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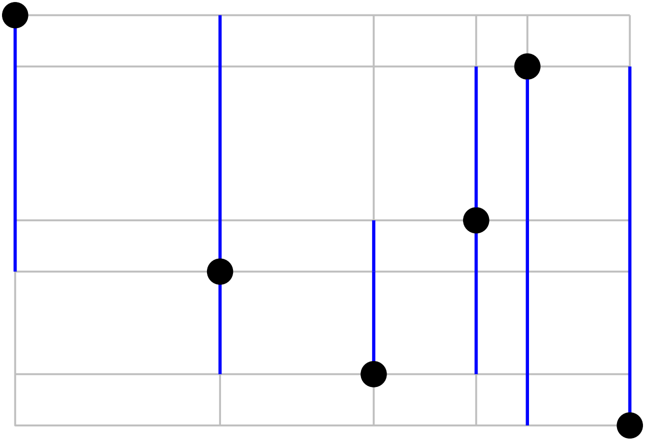

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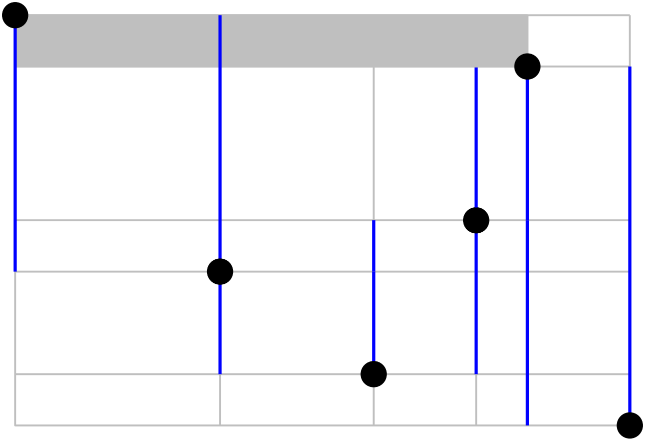

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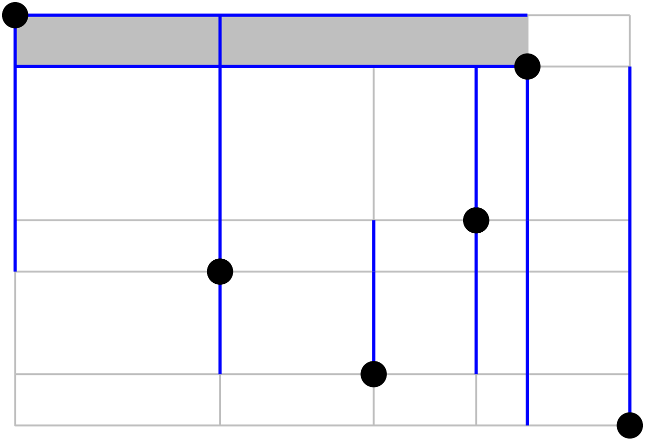

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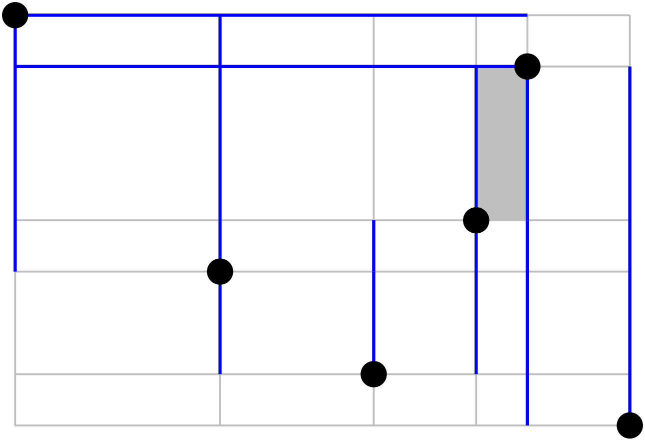

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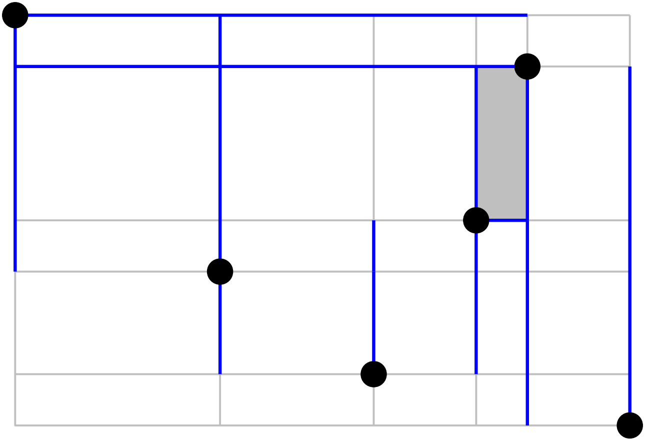

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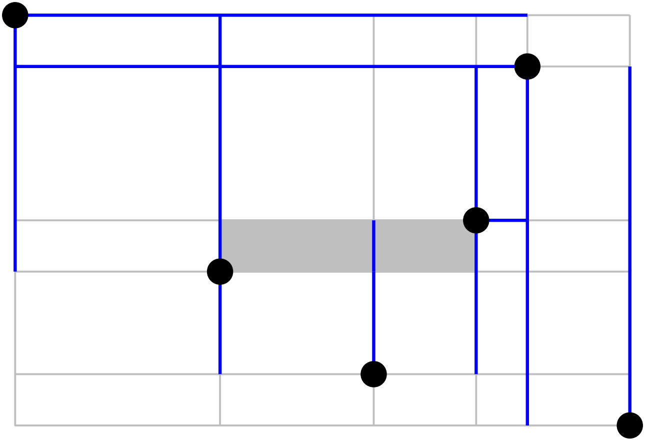

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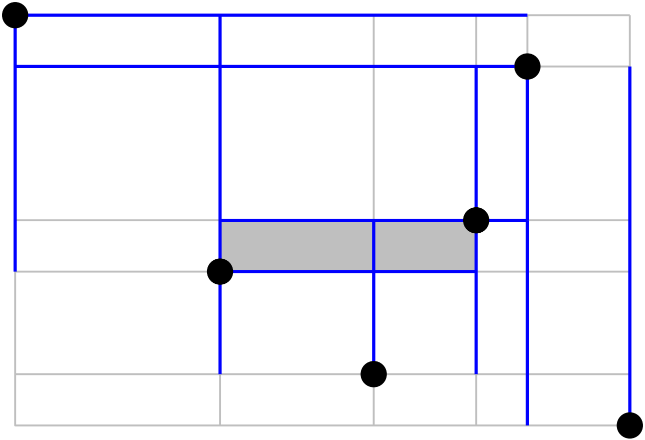

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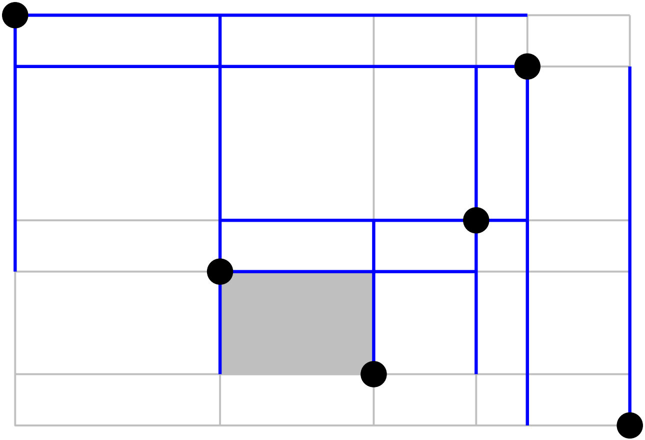

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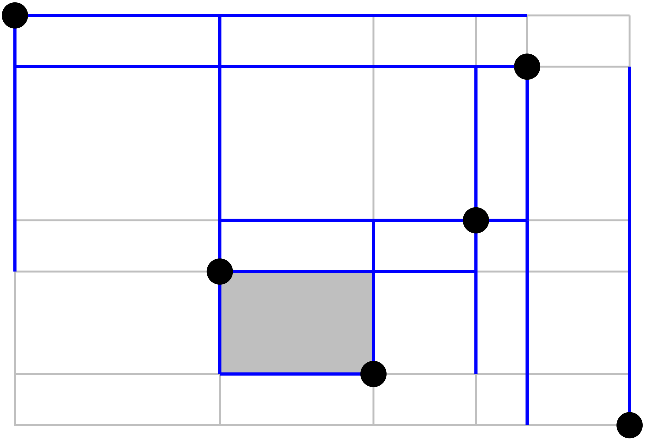

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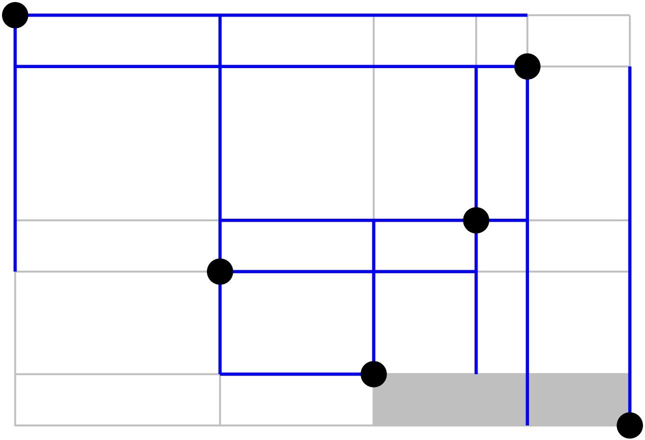

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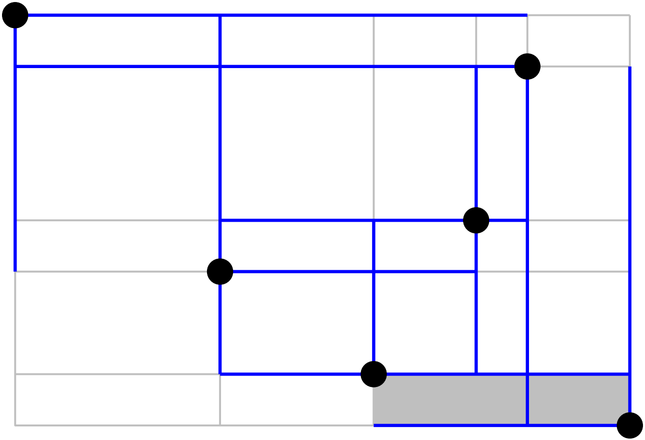

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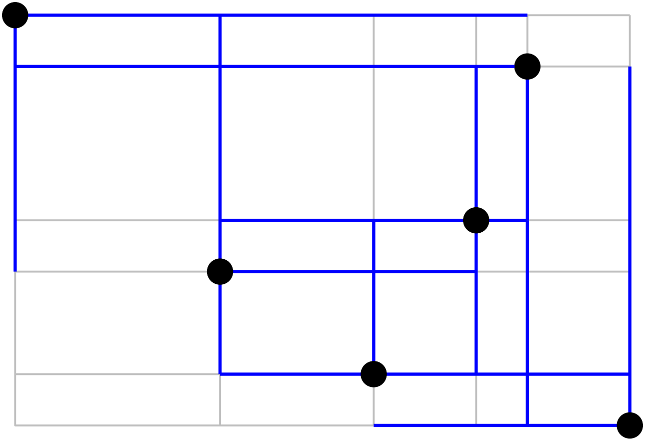
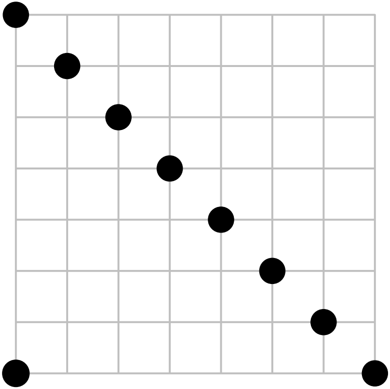


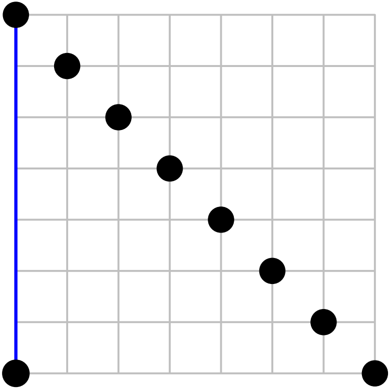
- Proceed likewise with *y-neighbors*.

After the sweep



- In general, no Manhattan network after sweep.

After the sweep



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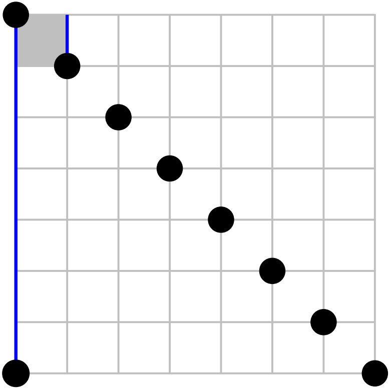

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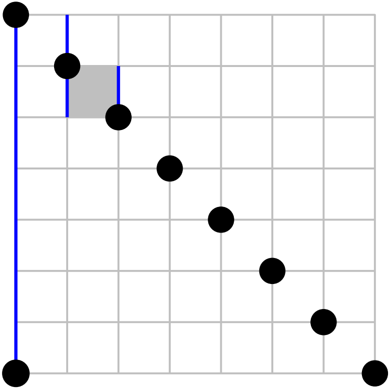

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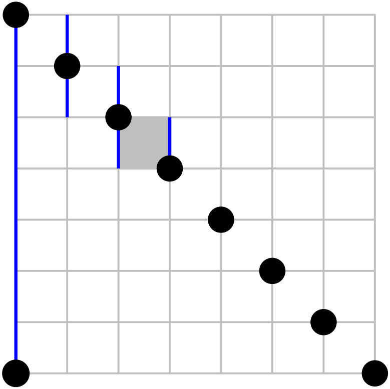

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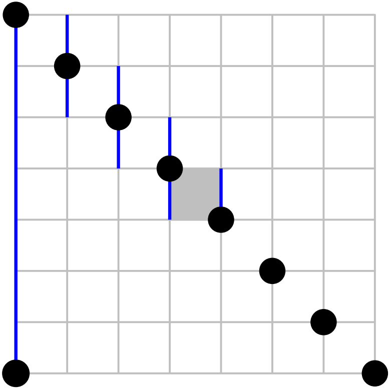

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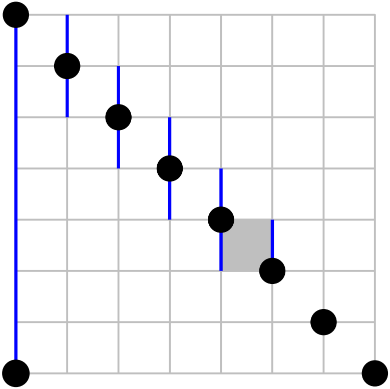

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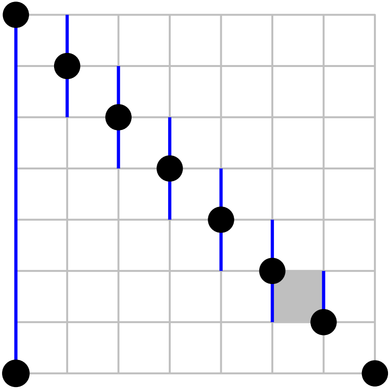

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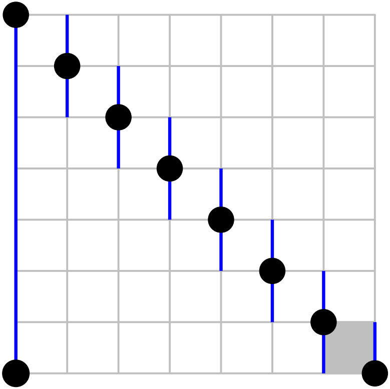

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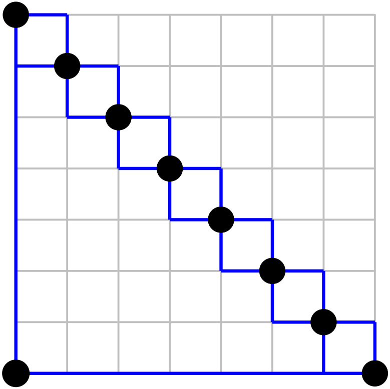

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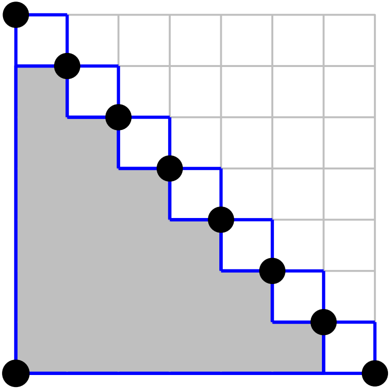

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- So-called **staircases** still empty.

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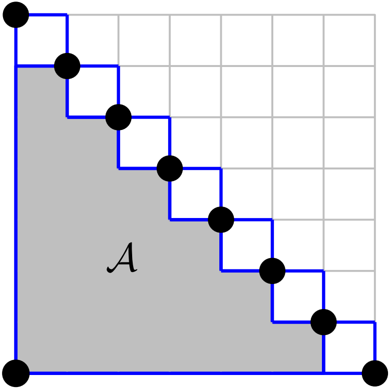
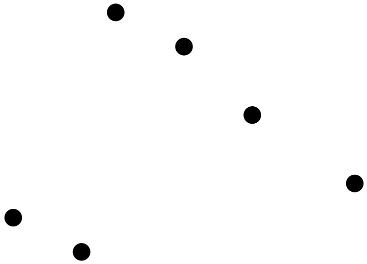


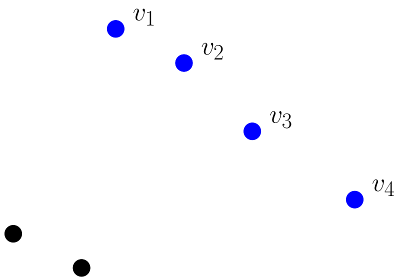
- In general, no Manhattan network after sweep.
- So-called **staircases** still empty.
- Call $\mathcal{A}$ the *staircase area*.

After the sweep



Definition (staircase)

After the sweep



Definition (staircase)

- k sequence points $(v_1, \ldots, v_k)$.

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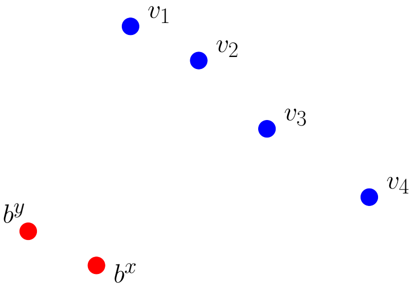

Definition (staircase)

- k sequence points $(v_1, \ldots, v_k)$.
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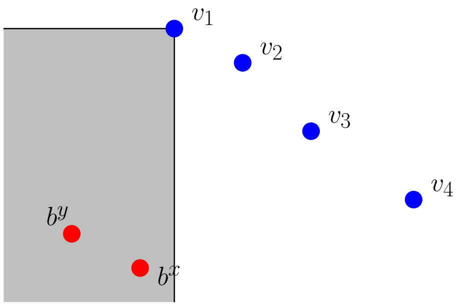

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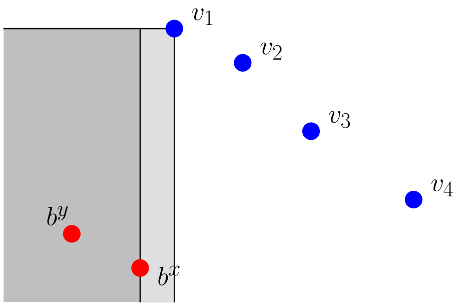

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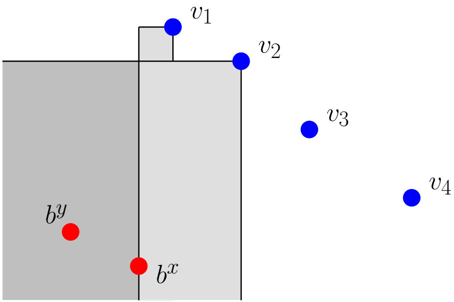

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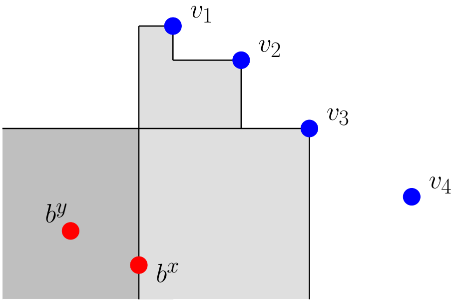

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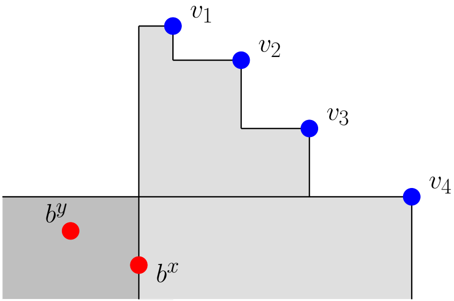

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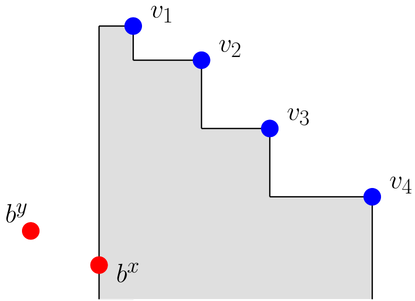

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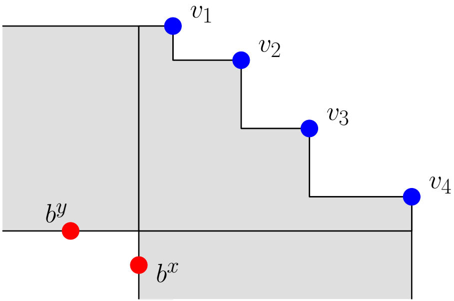

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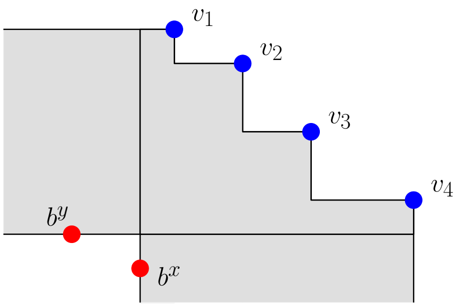
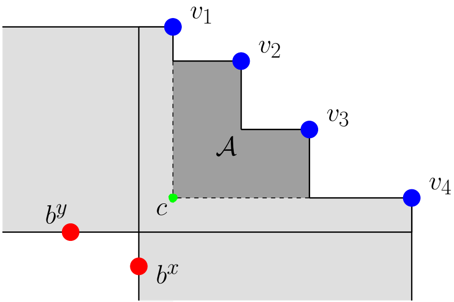


Observation

- The grey shaded areas contain no points.

After the sweep



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- The grey shaded areas contain no points.
- No sweep lines lie inside the staircase area $\mathcal{A}$.

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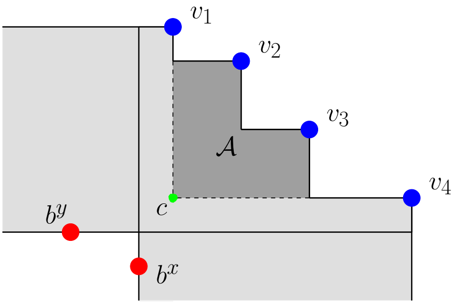
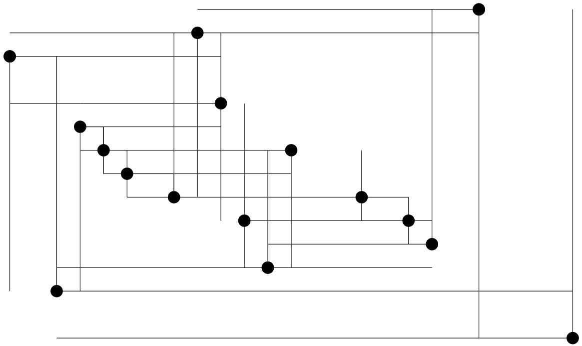


Observation

- The grey shaded areas contain no points.
- No sweep lines lie inside the staircase area $\mathcal{A}$.
- $\Rightarrow$ Points $v_3, \ldots, v_{k-2}$ need to be connected to *cross point* c.

After the sweep



Lemma

- *Except for staircase areas, all critical pairs of points are connected via shortest paths after the sweep.*

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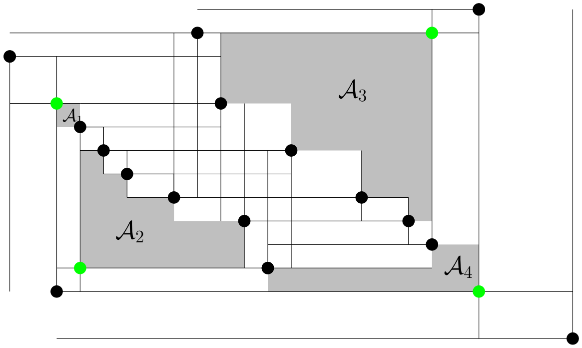

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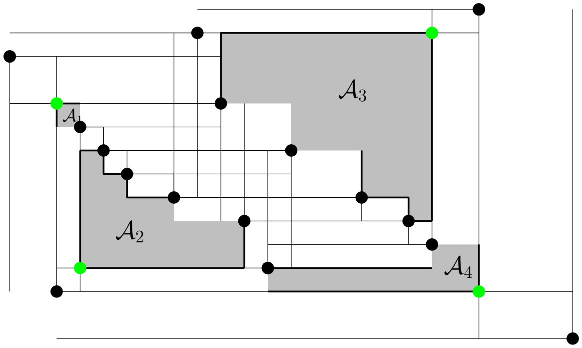

Lemma

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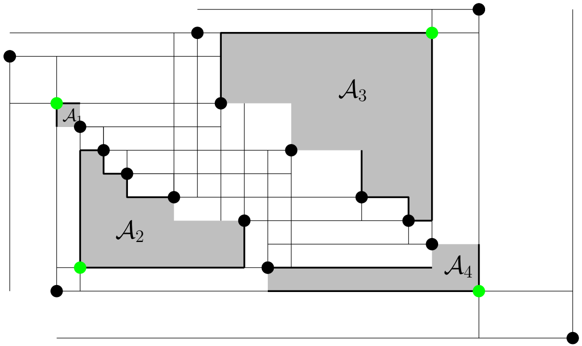
After the sweep



Lemma

- *Except for staircase areas, all critical pairs of points are connected via shortest paths after the sweep.*
- *The staircase areas $\mathcal{A}_i$ are bordered by as many line segments from the sweep step as possible, and are as small as possible.*

Approximation analysis

Consider the approximation seperately:

- inside staircases ($\mathcal{A} = \bigcup_i \mathcal{A}_i$, Phase II), and
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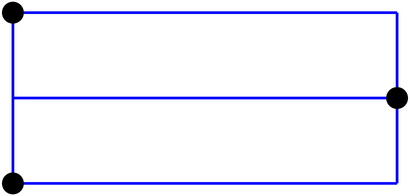

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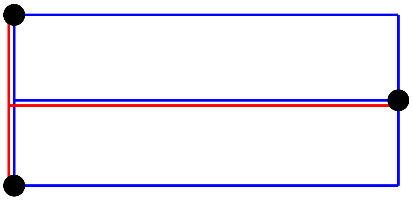

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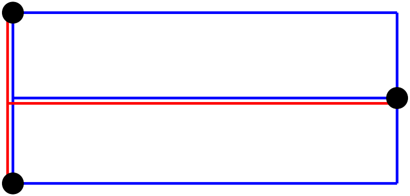
$\Rightarrow$ **3-Approximation.** ✓



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Phase II (Area $\overline{\mathcal{A}}$)

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- Phase II:

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- Altogether:

$$alg \;=\; alg_{\mathcal{A}} + alg_{\overline{\mathcal{A}}} \;\leq\; 2 \cdot opt_{\mathcal{A}^*} + 3 \cdot opt_{\overline{\mathcal{A}^*}} \;\leq\; 3 \cdot opt. \qquad \square$$

Conclusion

We presented a simplified 3-approximation algorithm for MMNs.

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Thank you!