Decomposing trees with large diameter

Romain Ravaux

May 2008

Definitions

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- Let $G = (V, E)$ be a n-vertex graph of order.
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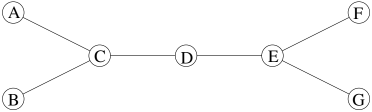

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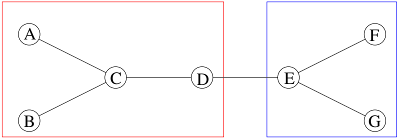
- 

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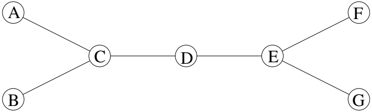
- 

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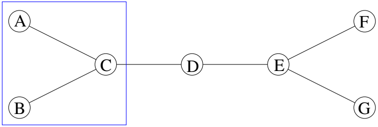
- 

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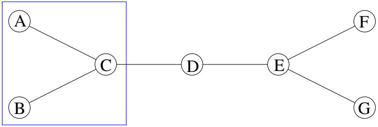
- 

- T is not λ-decomposable.
- A graph G is said decomposable if and only if for all partition λ of n the graph G is decomposable for λ.

Results

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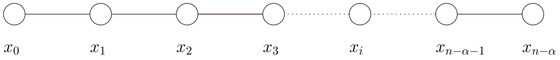
- 

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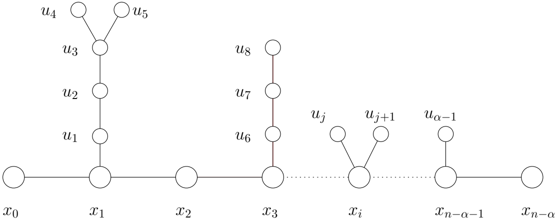
- 

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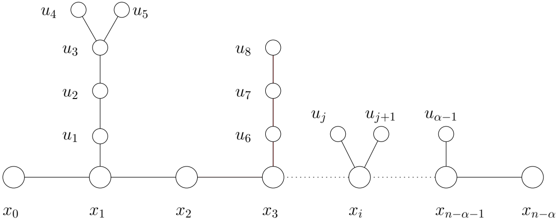
- 

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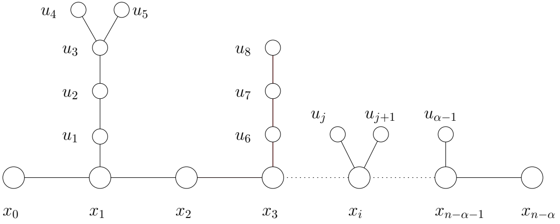

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- remark: number of partitions of n is $O(2^n)$.

Proof

- The spectrum of a partition $\lambda = (\lambda_1, \lambda_2, .., \lambda_p)$ is defined by $sp(\lambda) = \{\lambda_1, \lambda_2, \dots, \lambda_p\}$.

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Consider a n-vertex tree $T = (V, E)$ with diameter $D(T) = n - \alpha$. The tree T is decomposable for all partitions $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p)$ of n with $|sp(\lambda)| \geq \alpha$.

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Consider a n-vertex tree $T = (V, E)$ with diameter $D(T) = n - \alpha$. Deciding if the tree T is decomposable for all partitions λ with $|sp(\lambda)| < \alpha$ can be done with a time complexity $n^{O(\alpha)}$.

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- Deciding if T with diameter $n - \alpha$ is decomposable for a partition λ can be done with time complexity $n^{O(\alpha)}$.

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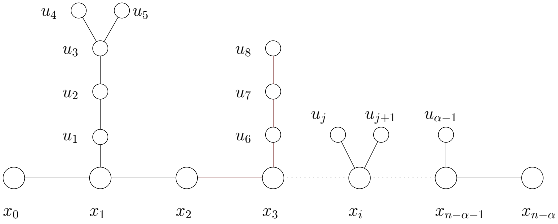
- 

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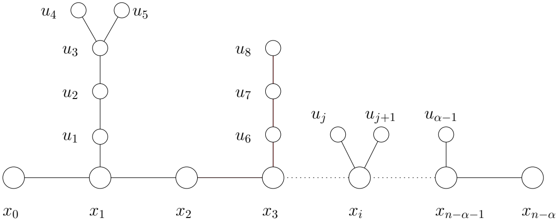
- 

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- Generate all possibles sets containing vertices $\{u_1, .., u_{\alpha-1}\}$.

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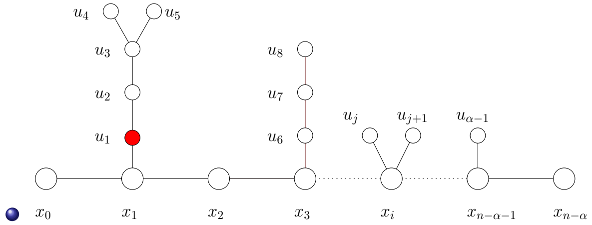

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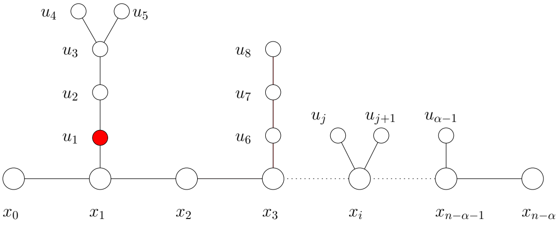
- 

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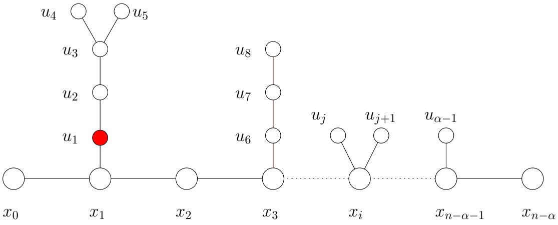
- 

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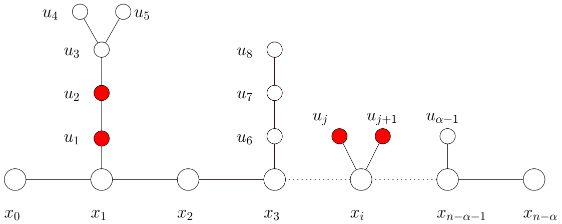

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- Example : Vertices u_2, u_j, u_{j+1} are in the set which contains vertice u_1.

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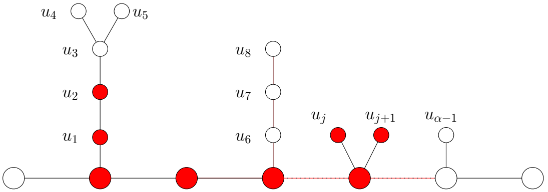

- x_0 x_1 x_2 x_3 x_i $x_{n-\alpha-1}$ $x_{n-\alpha}$
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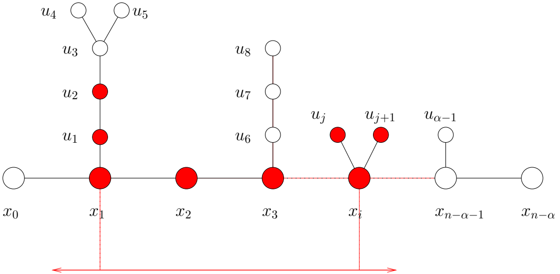

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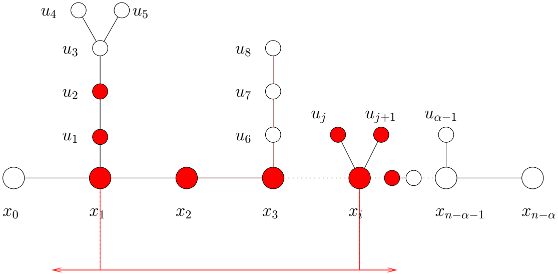

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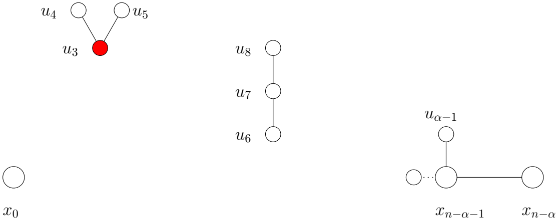
- 

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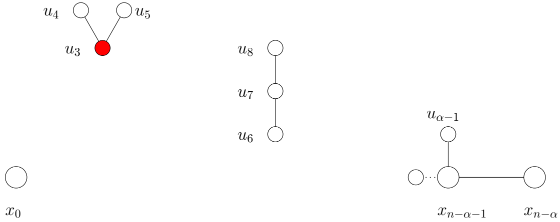

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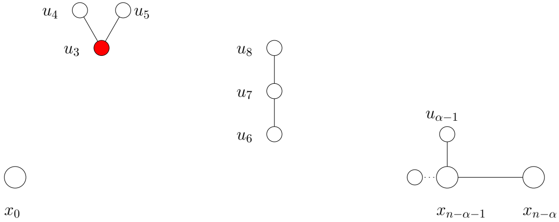
- 

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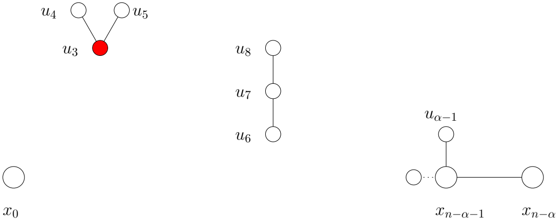
- 

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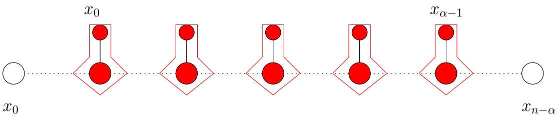
- 

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- $n^{O(\alpha)}$.

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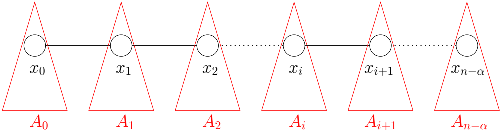
- 

- A_i is the set of vertices of the i^{eme} tree.

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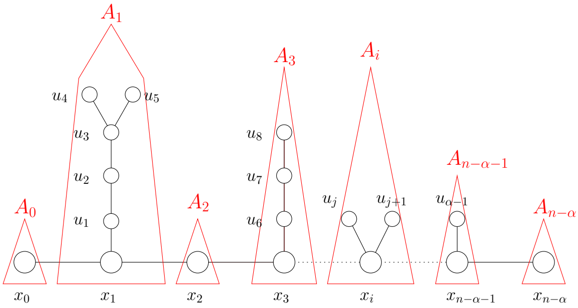
- 

- $A_1 = \{x_1, u_1, u_2, u_3, u_4, u_5\}$.

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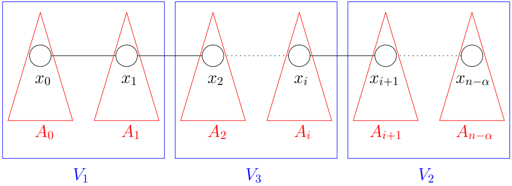
- 

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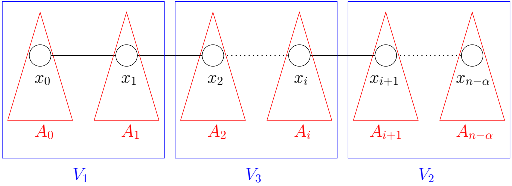

- $T = (V, E)$ with diameter $D(T) = n - \alpha$.
- $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p)$ of n with $|sp(\lambda)| \geq \alpha$.
- Show that it exists a (T,λ)-partition $V_1, V_2, .., V_p$ of V such that for any $0 \leq i \leq n - \alpha$ it exists $j \in \{1, .., p\}$ for which we have $A_i \subseteq V_j$.

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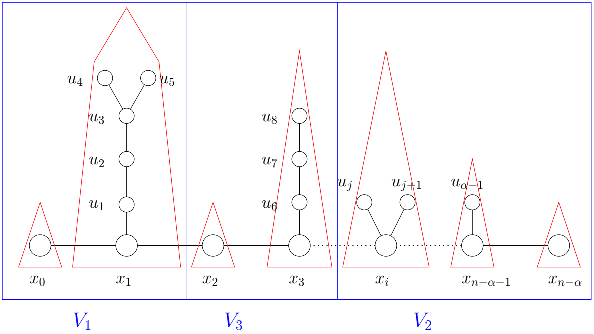
u_4
u_5
u_3
u_2
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u_8
u_7
u_6
u_j
u_{j+1}
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x_0
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x_2
x_3
x_i
$x_{n-\alpha-1}$
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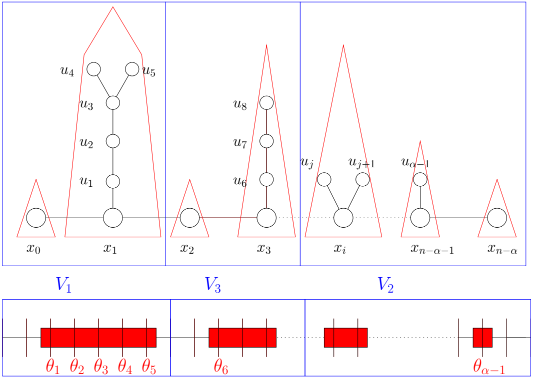
V_1
V_3
V_2

Proof



- 0 1 2 3 4 5 6 7 8 9 10 11 $n - \alpha$
- $I = \{\theta_1, .., \theta_{\alpha-1}\}$ the set of **forbidden integers**.
- $P = \{0, .., n\} - I$ set of possible integers.
- equivalent to show that it exists a permutation $\pi = \pi_1, .., \pi_p$ of $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_p)$ such that partial sums are in P.

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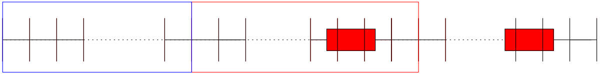
- other integers of λ: $(r_1, .., r_{p-\alpha})$, $R_i = \sum_{j=1}^{i} r_j$, and $R = R_{p-\alpha}$.
- T with diameter $D(T) = n - \alpha$, $I = \{\theta_1, .., \theta_{\alpha-1}\}$.

Proof

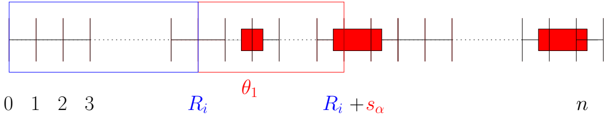
- Case 1: it exists $i \in \{0, .., p - \alpha\}$ such that $R_i < \theta_1$ and $R_i + s_\alpha > \theta_1$ with $R_i + s_\alpha \in P$



- 0 1 2 3 R_i θ_1 $R_i + s_\alpha$ n

Proof

- Case 2: it exists $i \in \{0, .., p-\alpha\}$ such that $R_i < \theta_1$ and $R_i + s_\alpha > \theta_1$ with $R_i + s_\alpha \in I$



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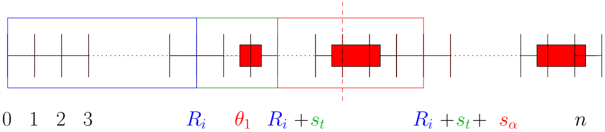

- it exists a $t \in \{1, .., \alpha - 1\}$ for which we have $R_i + s_t \in P$ and $R_i + s_t + s_{\alpha+1} \in P$

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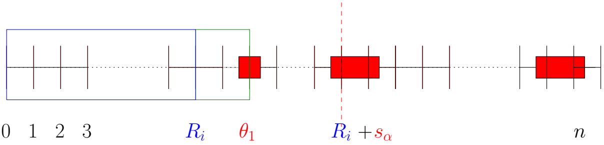
- 

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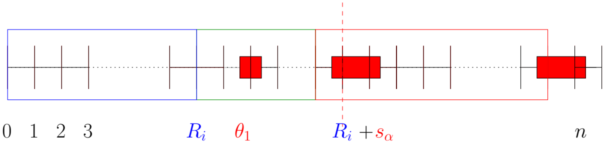
- 

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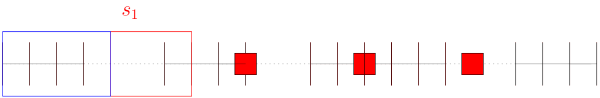
- Case 3: $R < \theta_1$ and $R + s_\alpha \leq \theta 1$



- $0 \quad 1 \quad 2 \quad 3 \quad R \qquad \theta_1 \qquad \theta_2 \qquad \theta_3 \qquad n$

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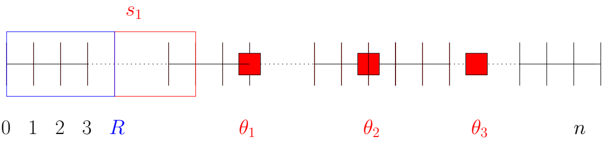
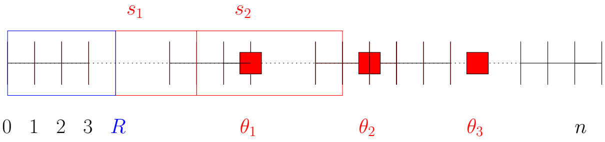


- $R + s_1 < \theta 1$

Proof

- Case 3: $R < \theta_1$ and $R + s_\alpha \leq \theta 1$
- Sub-case : It exists a $i \in \{2, .., \alpha - 1\}$ such that $R + S_{i-1} < \theta_{i-1}$ and $R + S_i \geq \theta_i$, with $R + S_{i-1} \in P$



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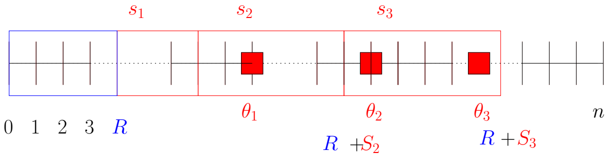
- 

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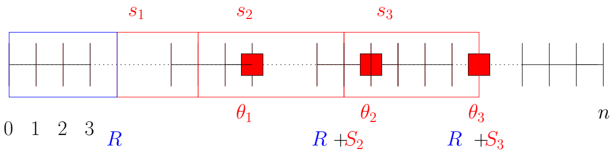
- 

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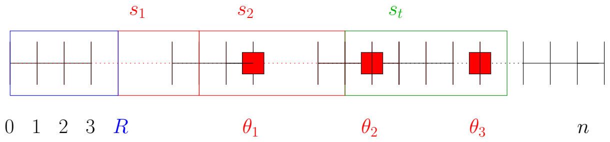

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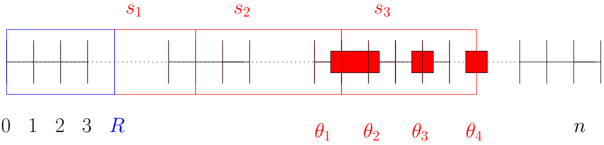

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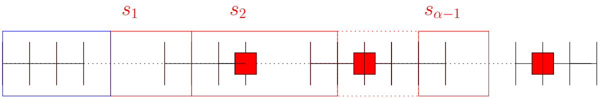
Proof

- Case 3: $R < \theta_1$ and $R + s_\alpha \leq \theta 1$
- Sub-case : It does not exist a $i \in \{2, .., \alpha - 1\}$ such that $R + S_{i-1} < \theta_{i-1}$ and $R + S_i \geq \theta_i$



- $0 \quad 1 \quad 2 \quad 3 \quad R \qquad \theta_1 \qquad \theta_2 \qquad \theta_{\alpha-1} n$

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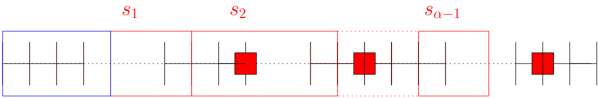

- $0 \quad 1 \quad 2 \quad 3 \quad R \qquad \theta_1 \qquad \theta_2 \qquad \theta_{\alpha-1} n$
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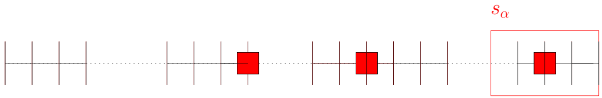

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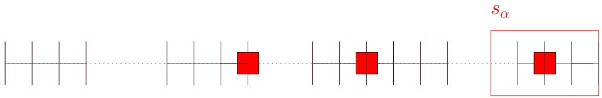

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Conclusion

- if α is a constant, Deciding if Tree T with large diameter $n - \alpha$ is decomposable is polynomial.

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- Which class in parametrized complexity ?