Radio Resource Allocation in OFDMA cellular network

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Summary

- OFDMA Cellular Network
- Allocation to RRes as an optimization pb.
- Complexity issues
- Network-flow models:
  - ✓ basic polynomial cases
  - ✓ convex cost model
- Heuristic approach and results
- Future directions

Orthogonal Frequency Division Multiple Access (OFDMA)

- Radio resource allocation: technique that assigns a subset of the available radio resources (e.g. power and bandwidth) to each user in the system according to a certain optimality criterion on the basis of the experienced link quality

- OFDMA is promising digital modulation technique for achieving high transmission rates in wireless communications
  - ✓ parallel data transmission of different users at the same time over different subcarriers
  - ✓ robustness to channel distortions
  - ✓ granular resource allocation capability

Radio resource allocation (OFDMA)

- Algorithms for multiuser OFDMA (Wong *et al.* IEEE JSAC '99)

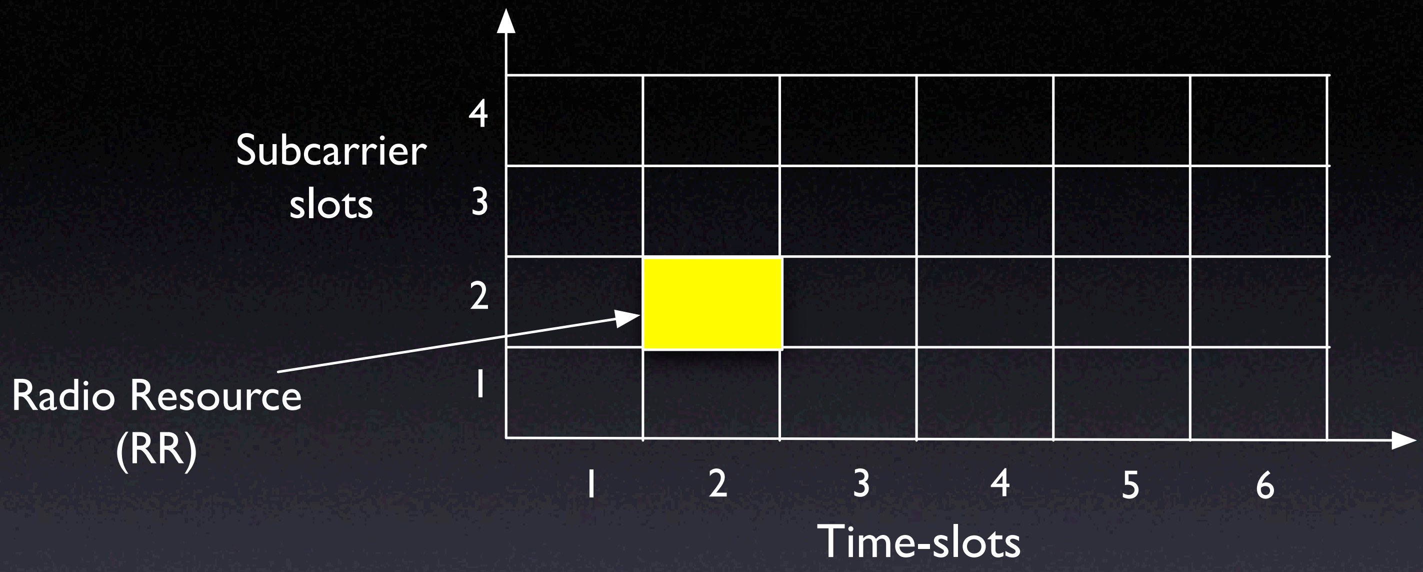
- <u>Single Cell Scenario</u>

  - Margin adaptive approach aims at minimizing the transmission power constraints on the user rate
    Kivanc *et al.* IEEE Trans. Wireless Comm. 2003,
    Kim *et al.* IEEE Trans. Veh. Tech. 2006

  - Rate adaptive approach aims at minimizing the user rate with a constraints on the transmission power
    Rhee, Cioffi 2000, Bohge *et al.* 2003

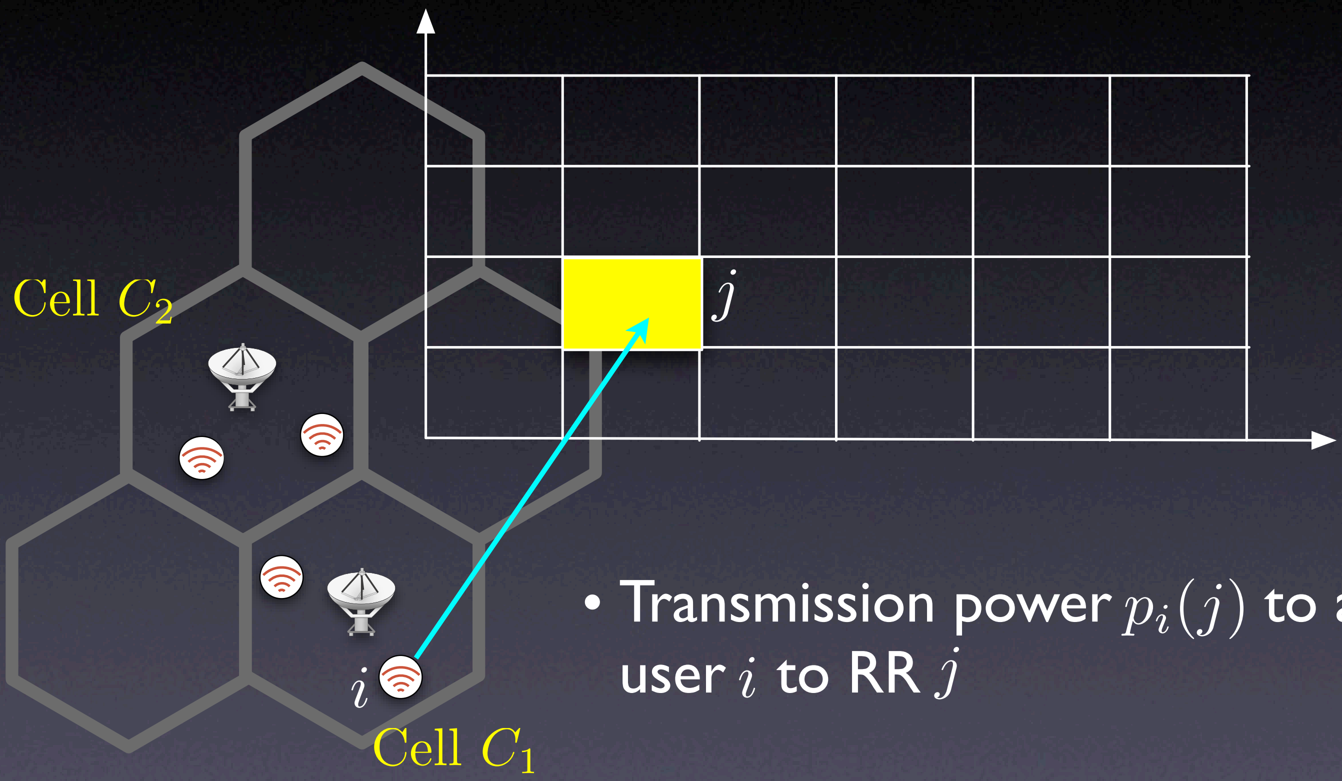
- <u>Multicell scenario</u>
  Li, Liu IEEE Trans. Wireless Comm. 2006,
  Gault *et al.* IEEE Trans. Comm. 2007

Packet Transmission



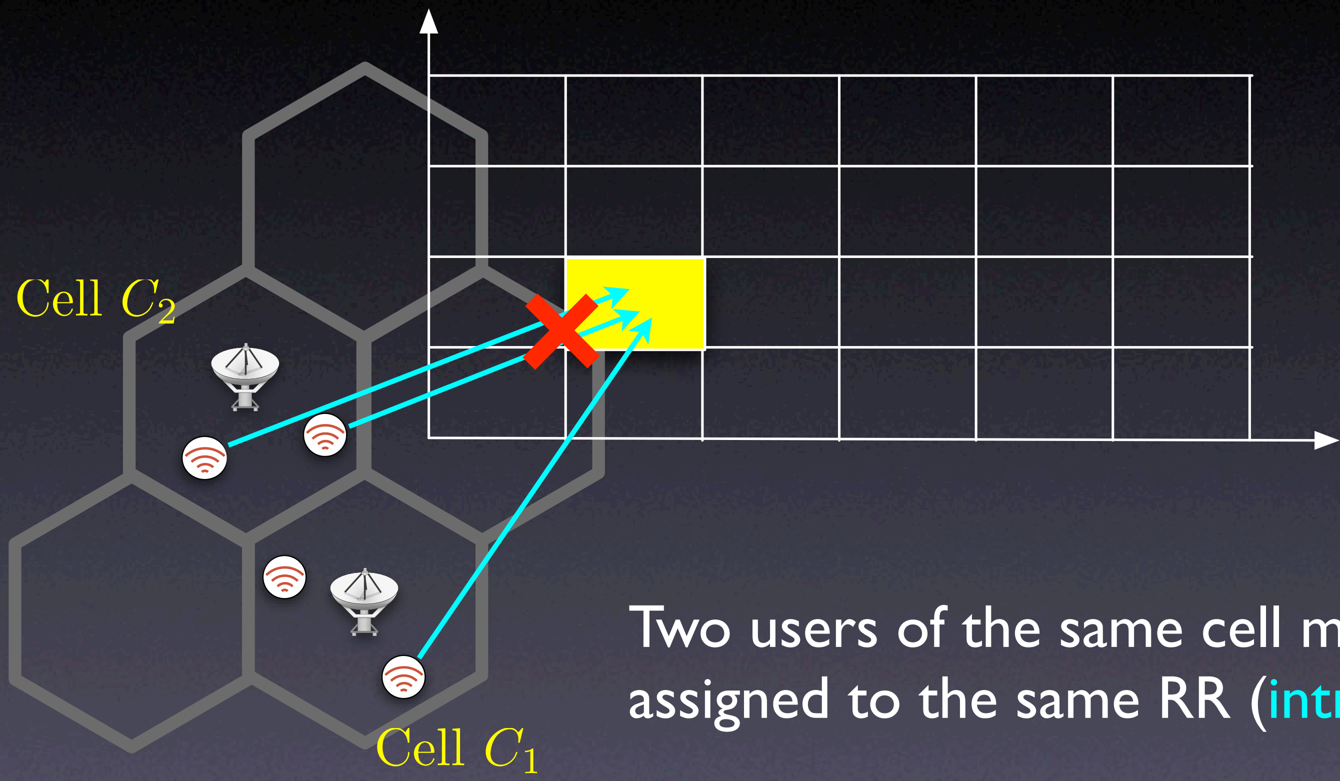
- Users' Data Packets are organized into radio frames for transmission
- An OFDMA radio frame is composed by a fixed number of Radio Resources (RRs)
- Each RR is a pair time-slot/subcarrier-slot

Radio-Resource allocation in OFDMA multi-cellular systems



- Transmission power $p_i(j)$ to assign user i to RR j

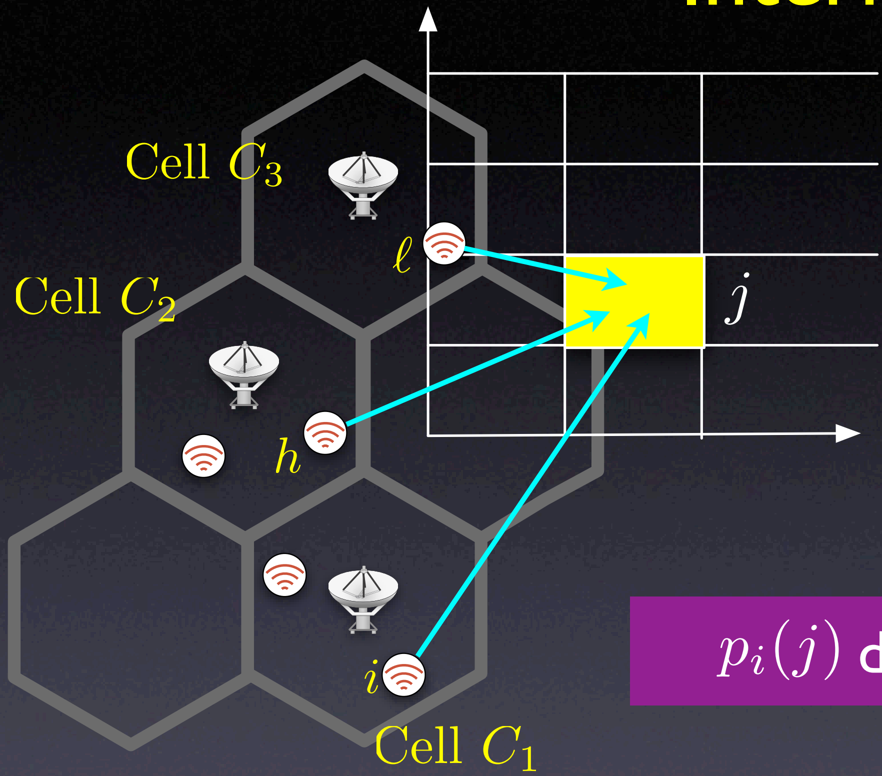
OFDMA multi-cellular systems
Interference phenomena



Two users of the same cell may not be assigned to the same RR (intra-cell interf.)

Transmission power $p_i(j)$ depends on the other users assigned to RR j (inter-cell interf.)

OFDMA multi-cellular systems
Interference phenomena



Inter-cell Interference

One user's transmission power depends on the transmission powers of the other users assigned to the same RR j :

$p_i(j)$ depends on $p_h(j)$ and $p_\ell(j)$

Transmission power $T(j)$ on RR j is the sum of the assigned users powers:

$$T(j) = p_i(j) + p_h(j) + p_\ell(j)$$

The Cellular RR allocation Problem (CRP)
Problem statement

Given:

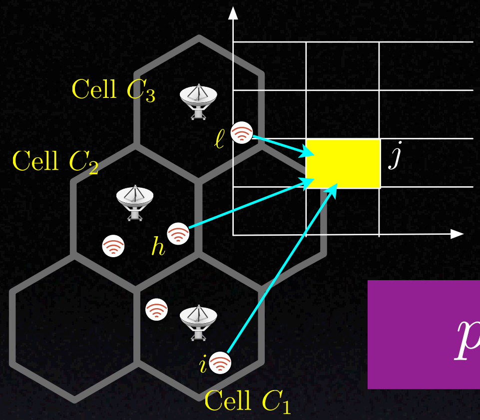
- radio frame with m radio resources (RRs);
- cells $C_1, \ldots, C_k$ with n_h users in cell h;
- transmission rate requirements (r_i RRs for user i);
- (cost model parameters)

Find:

for each RR j, the set $S(j)$ of users assigned to j

Such that:

- transmission rate to each user is provided;
- total transmission power $\sum_{j=1}^{m} T(j)$ is minimized



OFDMA multi-cellular systems
Interference cost models

$p_i(j)$ depends on $p_h(j)$, $p_\ell(j)$ ...

We consider three cost models:

I. Linear cost model

Let $S(j)$ be the set of users assigned to RR j, then

$T(j) = \sum_{i \in S(j)} p_i$
$Bp = A,\ p \geq 0_{|S(j)|}$ with $A,\ B$ given parameters

II. General cost model

$T(j)$ is a general function of set $S(j)$

III. Variable convex costs

$$T(j) = (\sum_{i \in S(j)} A_i) + f(|S(j)|)$$

The Cellular RR allocation Problem (CRP)
Linear cost model on RR j

Recall $p_i(j)$ depends on values $p_h(j)$ for all $h \in S(j)$

Given $S(j)$, the set of user assigned to RR j, the transmission powers $p_i(j)$ are given by sol., if any, of the foll. linear system

$$\left.\begin{aligned} p_i &= A_i + \sum_{\substack{h \in S(j) \\ h \neq j}} B_i^h p_h \\ p_i &\geq 0 \end{aligned}\right\} \forall i \in S(j)$$

$$A_i(j) = \frac{SIR_i B N_0}{G_i(j)} \qquad B_i^h(j) = \frac{SIR_i G_i^{b(\ell)}(j)}{G_i(j)}$$

are given data that accounts for target signal-interference ratio and channel gains

The Cellular RR allocation Problem (CRP)
Linear cost model on RR j

Recall $p_i(j)$ depends on values $p_h(j)$ for all $h \in S(j)$

Given $S(j)$, the set of user assigned to RR j, the transmission powers $p_i(j)$ are given by sol., if any, of the foll. linear system

$$\left.\begin{array}{l} p_i = A_i + \sum_{\substack{h \in S(j) \\ h \neq j}} B_i^h p_h \\ p_i \geq 0 \end{array}\right\} \forall i \in S(j)$$

- $A_i(j)$ is the fixed cost of assigning only user i to RR j
- $B_i^h(j)$ is the variable cost coefficient depending on the inter-cell interference experienced by user i when user h transmits on RR j

CRP Complexity Issues

Cells		Compl.ty
1	single cell, ltd. power, identical $B_i^h(j)$	un. NP-hard
_	identical $A_i(j)$	un. NP-hard
_	convex variable cost model	polynomial
2	more general costs	un. NP-hard
2	linear model costs	open
2	identical resources	polynomial
k fixed	identical users	polynomial
3	linear model costs	un. NP-hard

Thm.: CRP with 3 cells is NP-hard
(under linear cost model)

From 3-Dim. Axial Assignment with decomposable costs (Burkard et al. 96)

Given 3 sets of integers

$$A = \{a_1, \ldots, a_n\};\ B = \{b_1, \ldots, b_n\};\ C = \{c_1, \ldots, c_n\};$$

find 2 permutations π and ρ of B and C such that

$$\sum_{i=1}^{n} a_i b_{\pi(i)} c_{\rho(i)}$$

is mimimum.

With 3 cells and n users per cell $\Rightarrow$ triples of users on each cell

Obs.: CRP with 2 cells is NP-hard
(under general cost model)

Trivial from 3-Dim. Matching

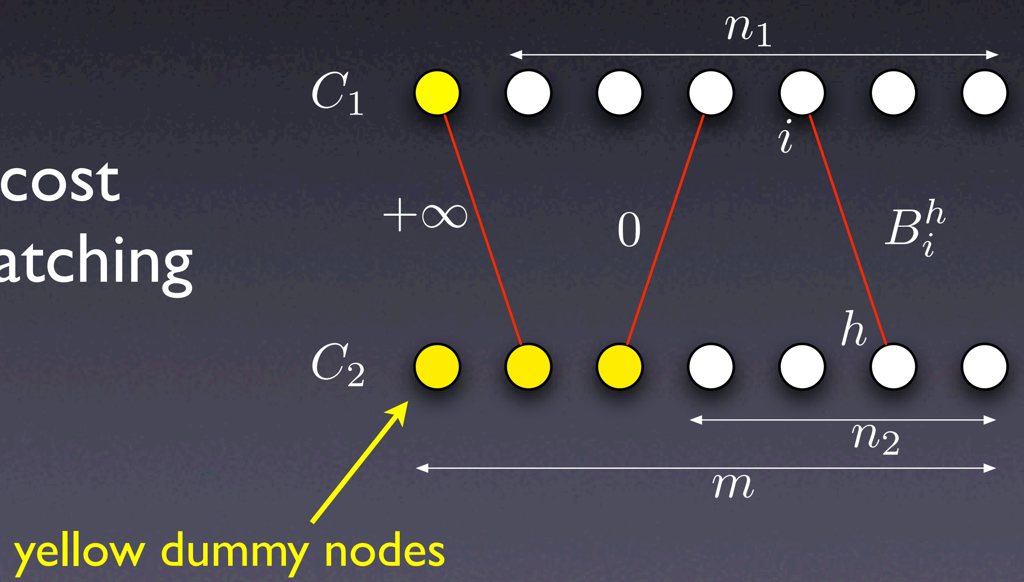
here the triple is: (i) user from one cell, (ii) user from the other cell, and (iii) RR

Basic polynomial cases:
2 cells, m identical radio-resources

(i.e. $A_i(j) = A_i;\ B_i^h(j) = B_i^h$)

feasible sol. iff $m \leq n_1 + n_2 \leq 2m$

minimum cost
perfect matching



Basic easy cases:
2 cells, $n_1 + n_2$ identical users

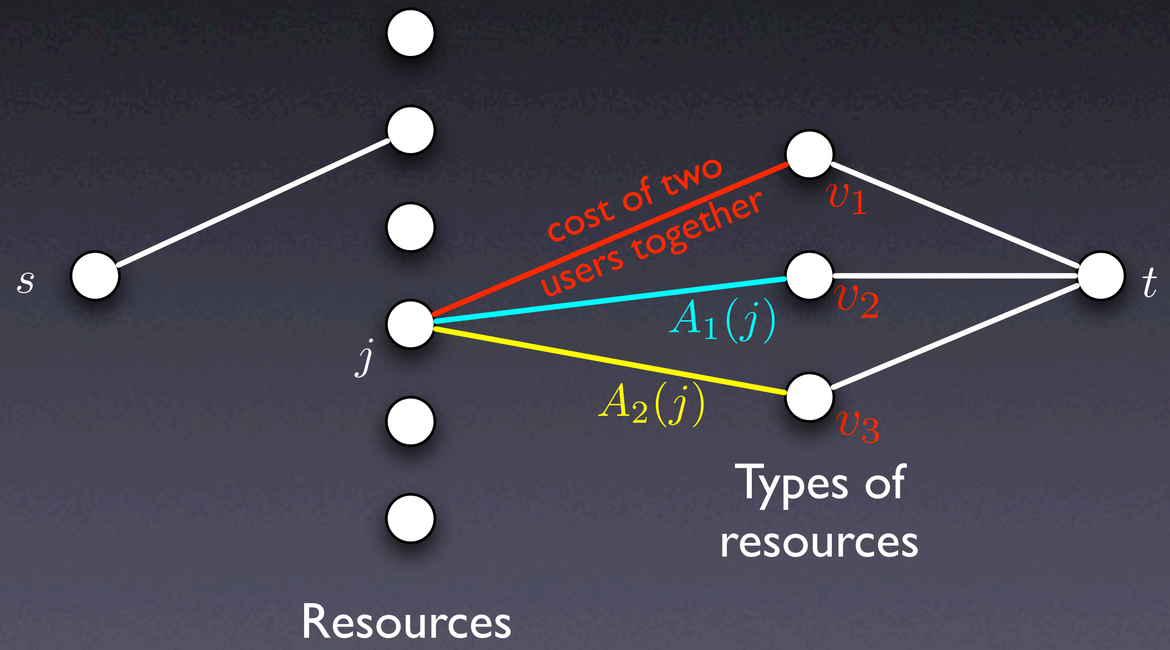
Cost depends on the # of users on a resource

Network Flow model (based on 3 types of res.)

- $n_1 + n_2 - m$ res. with 2 users : node v_1
- $m - n_1$ with one user of C_1: node v_2
- $m - n_2$ with one user of C_2: node v_3

Basic easy cases:
2 cells, $n_1 + n_2$ identical users (cont.d)

Min cost Flow model



A simplified CRP model

convex variable costs on RR j

Given $S(j)$, the set of user assigned to RR j, consider the foll. simplified model for transmission power at RR j :

$$T(j) = \sum_{i \in S(j)} p_i = \underbrace{\sum_{i \in S(j)} A_i}_{\text{fix. cost}} + \underbrace{f(|S(j)|)}_{\text{var. costs}}$$

where $f(\cdot)$ is strictly convex increasing

$$f(n+1) - f(n) > f(n) - f(n-1) \quad n \in \mathbb{Z}_+$$

(adding one more user is more costly!)

A simplified CRP model
convex variable costs on RR j

Marginal variable cost at a resource is increasing:

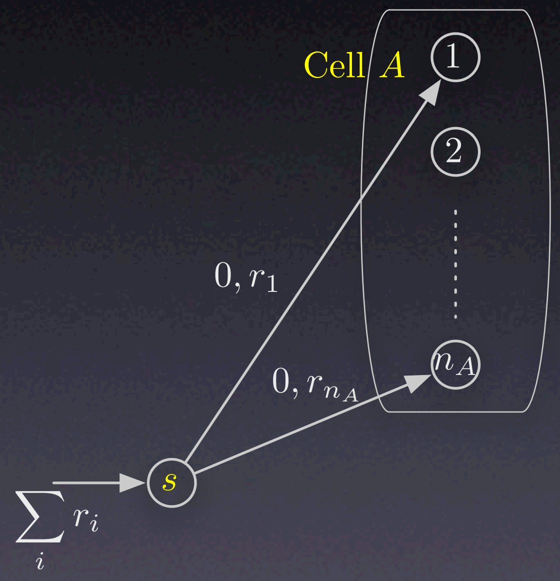
$$\Delta_n = \begin{cases} 0 & n = 0 \\ f(n) - f(n-1) & n \geq 1 \end{cases}$$

Because of $f(\cdot)$, adding one more user is more costly: i.e.:

$$\Delta_{n+1} > \Delta_n \quad n \in \mathbb{Z}_+$$

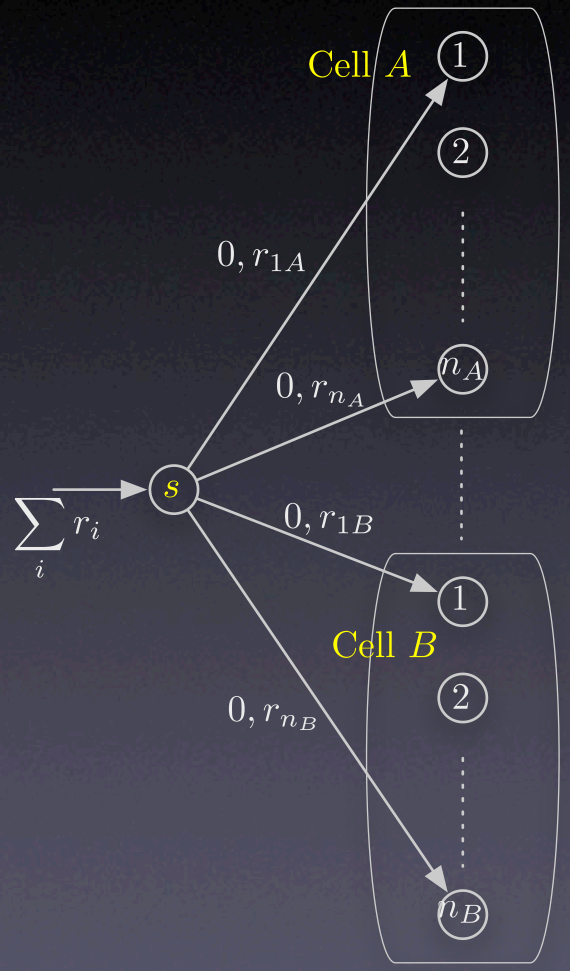
A simplified CRP model: convex variable costs
Network Flow Model

Labels at arcs are $(\text{cost}, \text{capacity})$

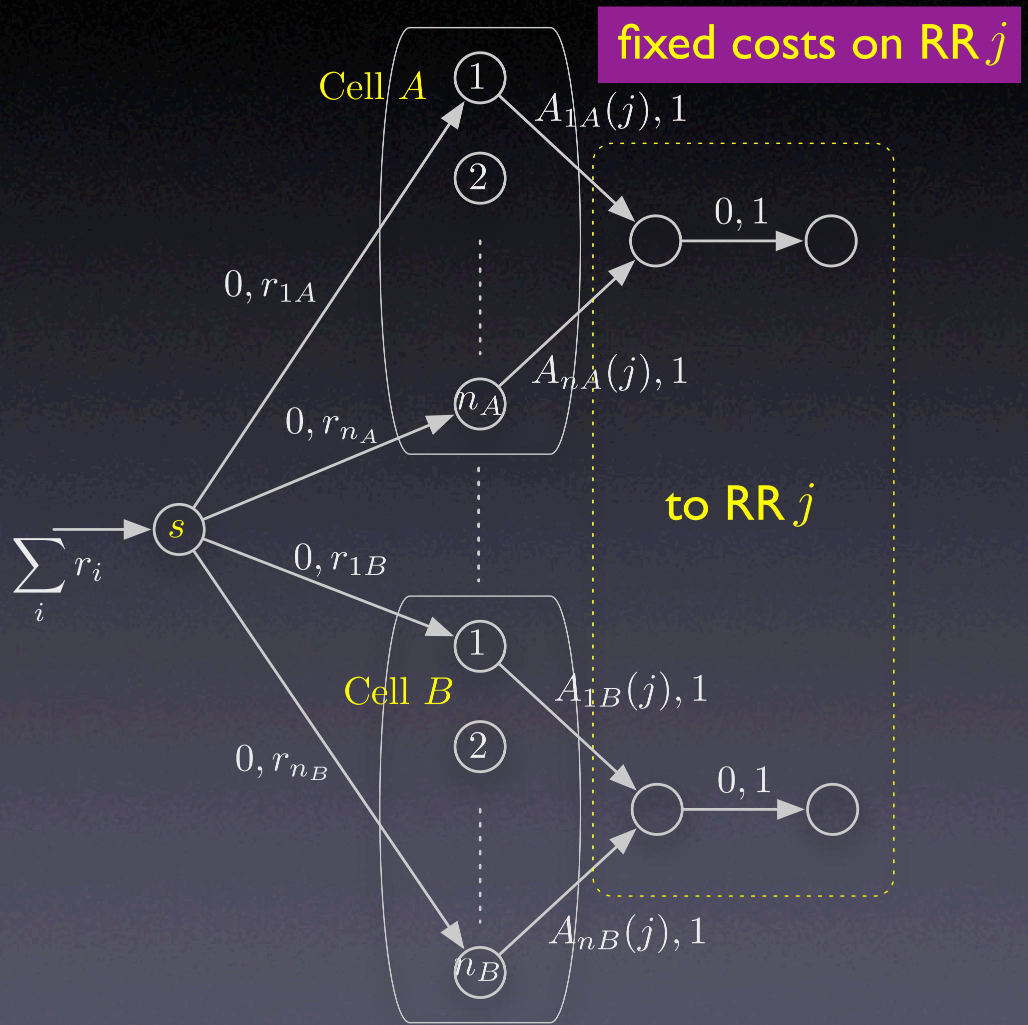


A simplified CRP model: convex variable costs

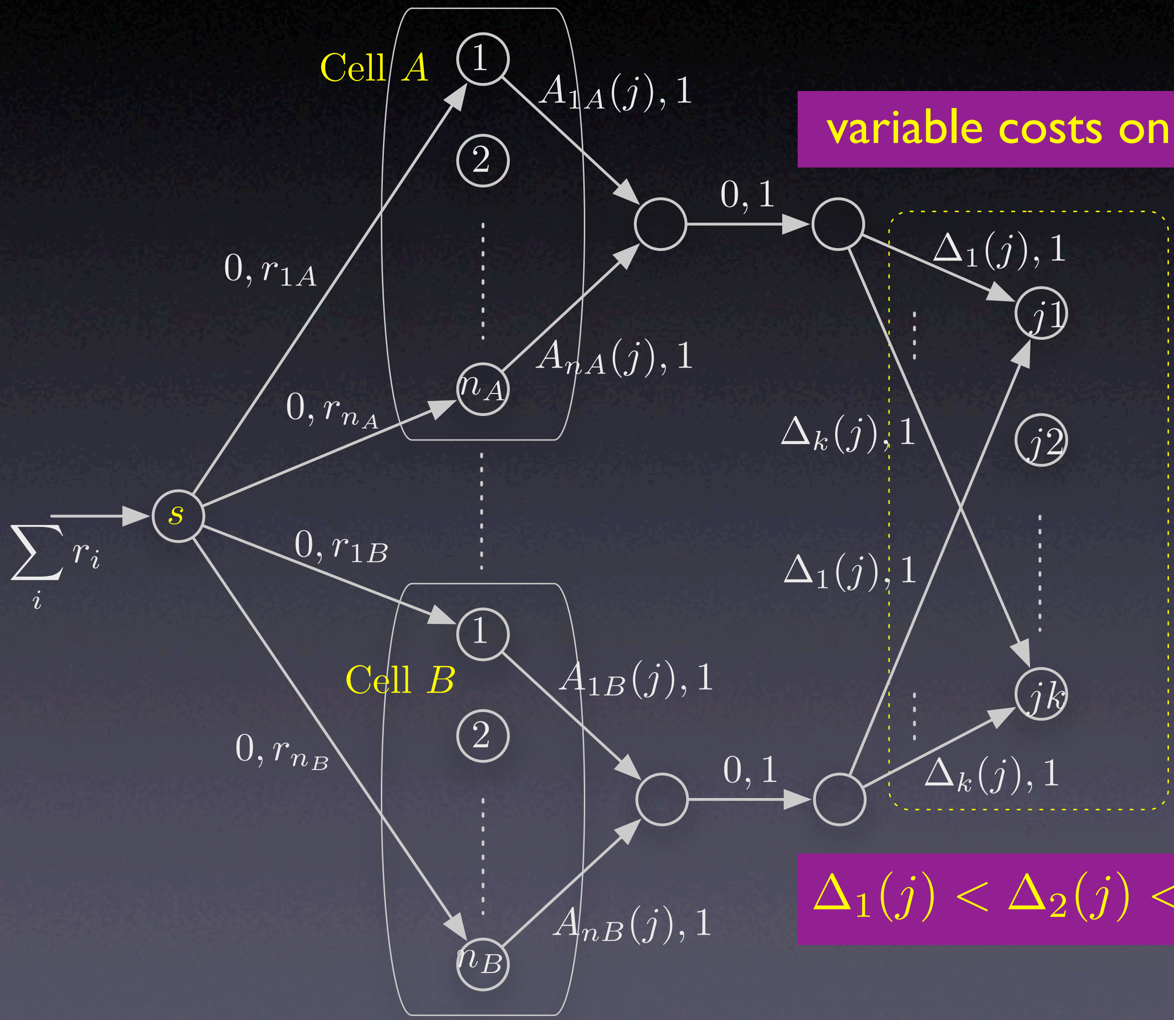
Network Flow Model



A simplified CRP model: convex variable costs
Network Flow Model



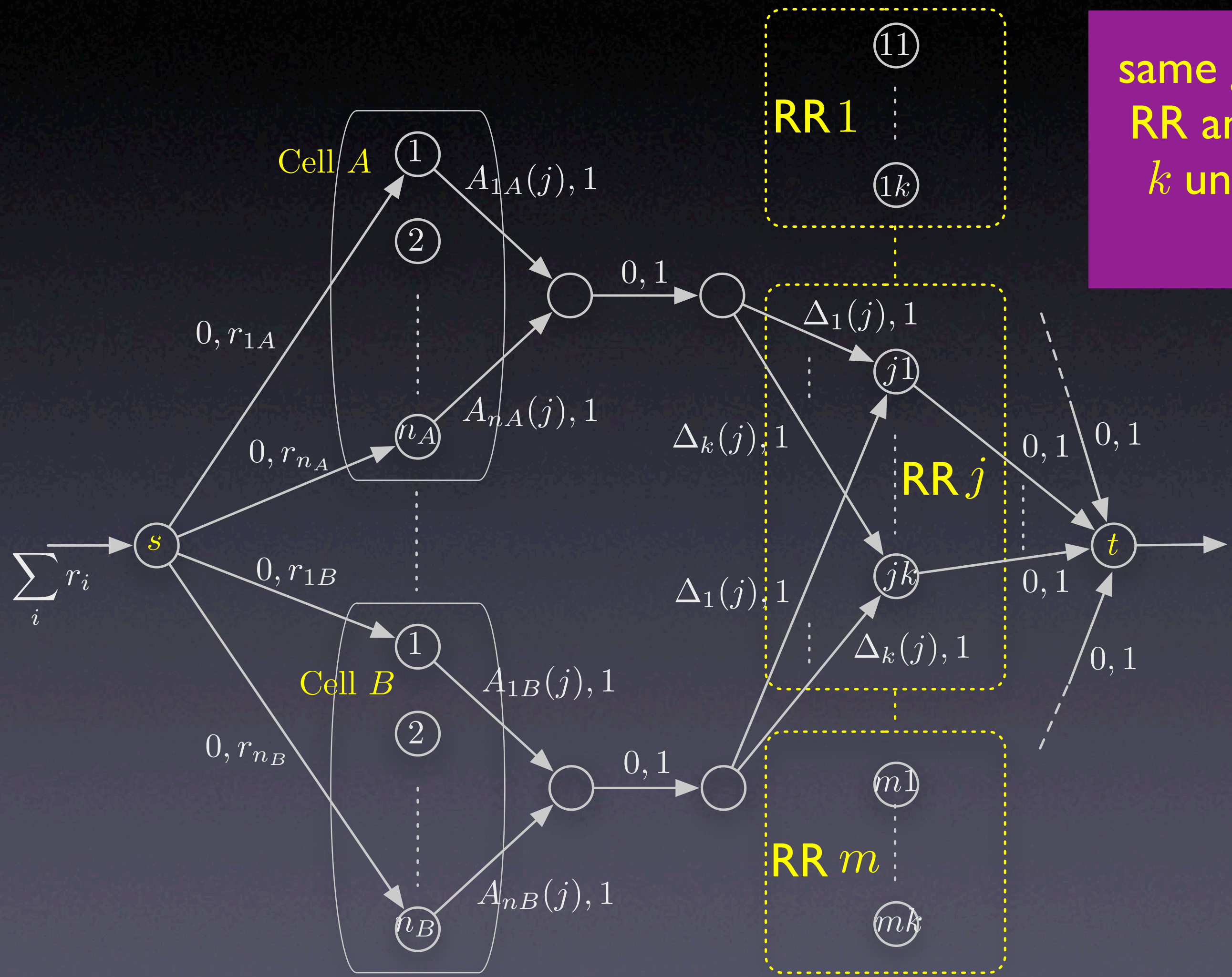
A simplified CRP model: convex variable costs
Network Flow Model



variable costs on RR j

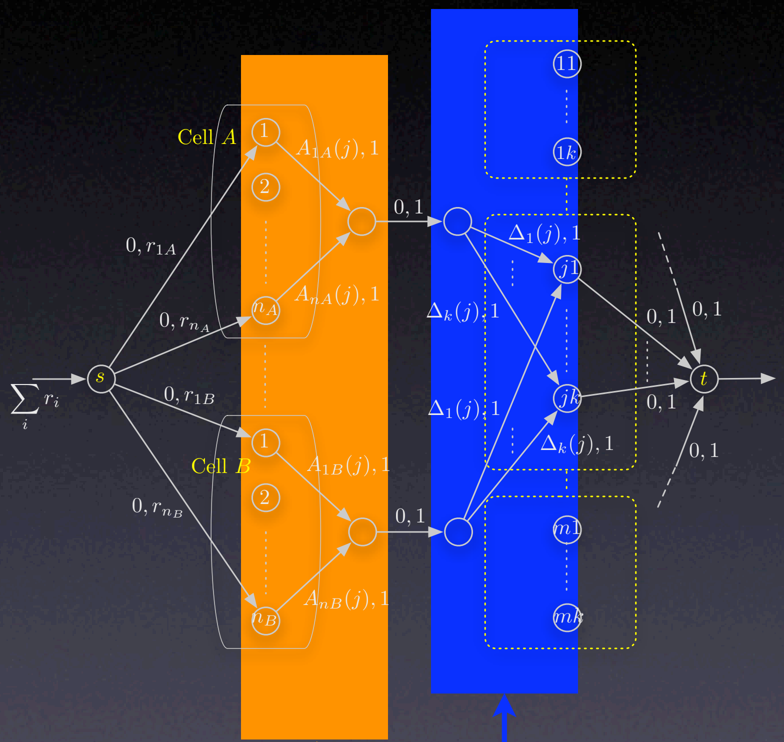
$$\Delta_1(j) < \Delta_2(j) < \ldots < \Delta_k(j)$$

A simplified CRP model: convex variable costs
Network Flow Model



same gadget for every RR and send at most k units for each RR to a sink

A simplified CRP model: convex variable costs
Network Flow Model



Observation: there is a biunique correspondence between any <u>integer flow</u> on the network and a feasible solution of CRP (under the convex variable cost model)

variable costs part

fixed costs part

Heuristic for CRP

based on the convex variable costs flow model

- Idea: to approximate the costs of CRP in order to fit the convex variable cost model:
  - ✓ if we set $A_i(j) \longleftarrow a(j)$ and $B_i^h(j) \longleftarrow b(j)$ for all users i and RRs j :
  - ✓ the transmission power at RR j when n users are assigned to it, becomes:

$$T'_j(n) = \underbrace{na(j)}_{\text{fixed costs}} + \underbrace{\frac{na(j)(n-1)b(j)}{1-(n-1)b(j)}}_{\text{convex variable costs}}$$

  - ✓ use the network flow model to obtain a users-RRs assignment

Heuristic for CRP

based on the convex variable costs flow model

- Choose initial values of $a(j),\ b(j)$ (average / min.)
- Run the netw. flow model
- Obs.: the assignment $S(j)$ thus obtained may not be feasible (negative powers)
  - ✓ if feasible assignment: improve solution quality by updating values of $a(j),\ b(j)$
  - ✓ if unfeasibility is detected: try (various) search schemes or remove users

Computational experiments
heuristic vs. ILP (exact) approach

- Test the net-flow heuristic against (natural) ILP on a set of 100 instances
  - ✓ 7 cells, 28 users, 4 users per cell;
  - ✓ $m = 16$ Radio Resources;
  - ✓ spectral efficiency values $\eta_i = 2$ for all users i
  - ✓ $\eta_i = \log_2(1 + SIR_i)$, and determines the fixed and variable costs $A_i(j), B_i^h(j)$
  - ✓ r_i is constant for all users i
- Algorithm coded in C,
- CPLEX 9.1 to solve the network flow problems
- 1.6 GHz Pentium M laptop, 1 GB RAM

Computational experiments

heuristic vs. ILP (exact) approach

Instance nr.s	100	85	70
CPLEX trunc.			
Avr. power	48.56		
Avr. CPU time	201.68 sec		
Avr. Infeasib. at RR	0	0	0
Net-Flow Heur.			
Avr. power	76.74 (+36%)	+22% gap	+14%
Avr. CPU time	0.285 sec.	0,281 sec.	0,279 sec.
Avr. Infeasib. at RR	0.015	0	0

Conclusions
(some) future directions

- Complexity issue: open case (2 cells, linear cost model)
- Improvement of the net-flow heuristic and code optimization
  - ✓ new experiment scenario
  - ✓ faster local search
- Design of distributed heuristic