Maximum Profit Wavelength Assignment in WDM Rings

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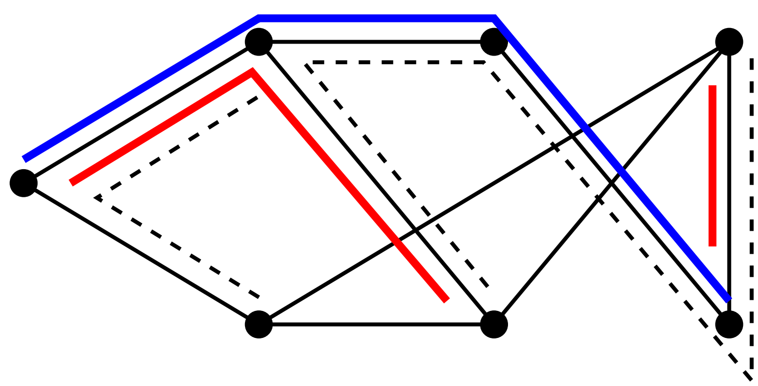
Transparent WDM networks

- Wavelength Division Multiplexing (WDM)
  - several “channels” per fiber
- Transparent routing
- Restrictions:
  - requests using the same edge have different frequencies (colors)
  - wavelength continuity
- Requests may have different profits

Maximum profit path coloring

Def. MaxPr-PC problem:

- input: graph G, path set P, # colors k, profits $w : P \to \mathbb{Q}$
- solution: a k-colorable subset of paths $P' \subseteq P$
- goal: maximize $w(P') = \sum_{p \in P'} w(p)$



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Remarks:

- NP-hard in rings and trees
- polynomial-time solvable in chains [CL95]

Related work

On the cardinality version (MaxPC):

- Iterative algorithm [WL98], $\rho \approx 1.58$
- “Combine” [NPZ03], $\rho = 3/2$
- Iterative + local search [Car07], $\rho = 4/3$

On MaxPr-PC:

- MaxPr-PC with routing [LLWZ05], $\rho = 2$
- Adaptation of iterative algorithm, $\rho \approx 1.58$
- LP + randomized rounding [Car07], $\rho \approx 1.49$

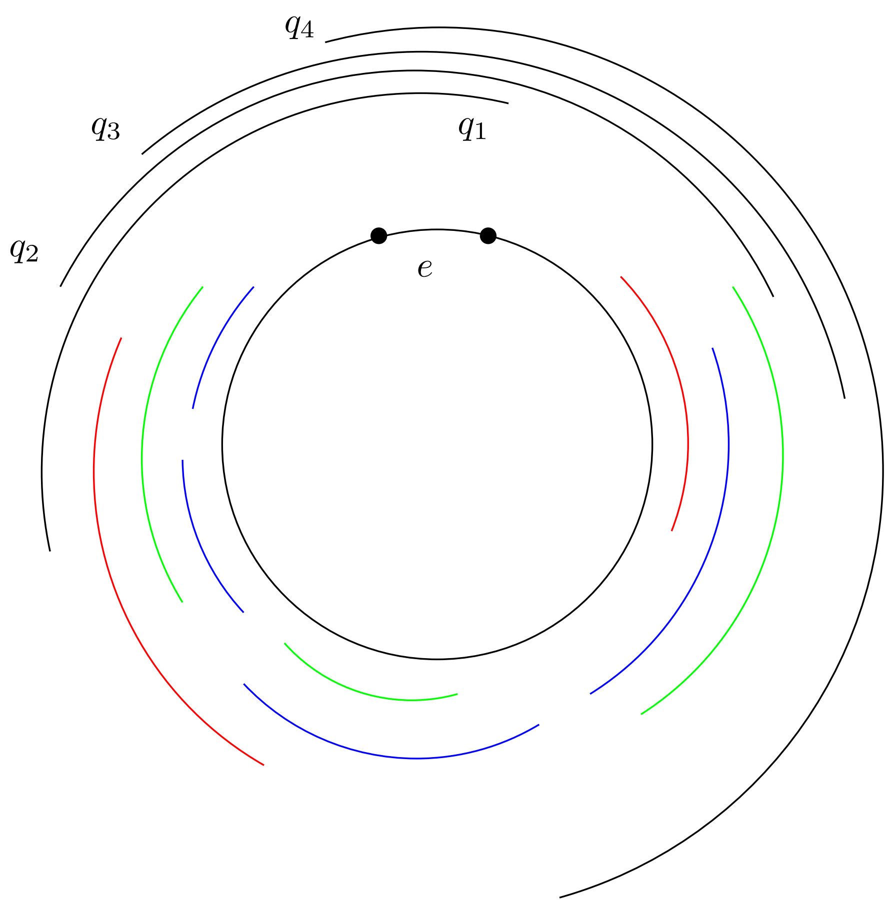
In the rest of this talk...

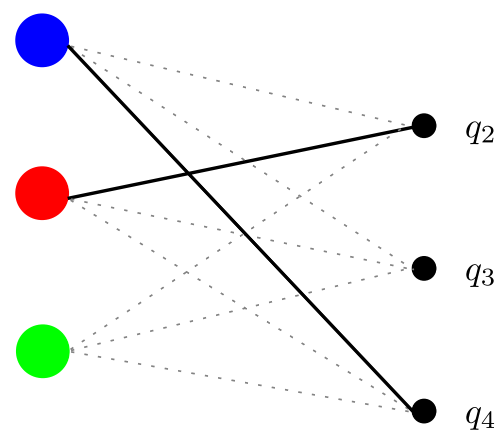
- Match and replace
  - a fast, combinatorial 2-approximation algorithm for MaxPr-PC in rings
- Tradeoffs between running time and approximation guarantee
  - some experimental results
- Further work

Match and replace

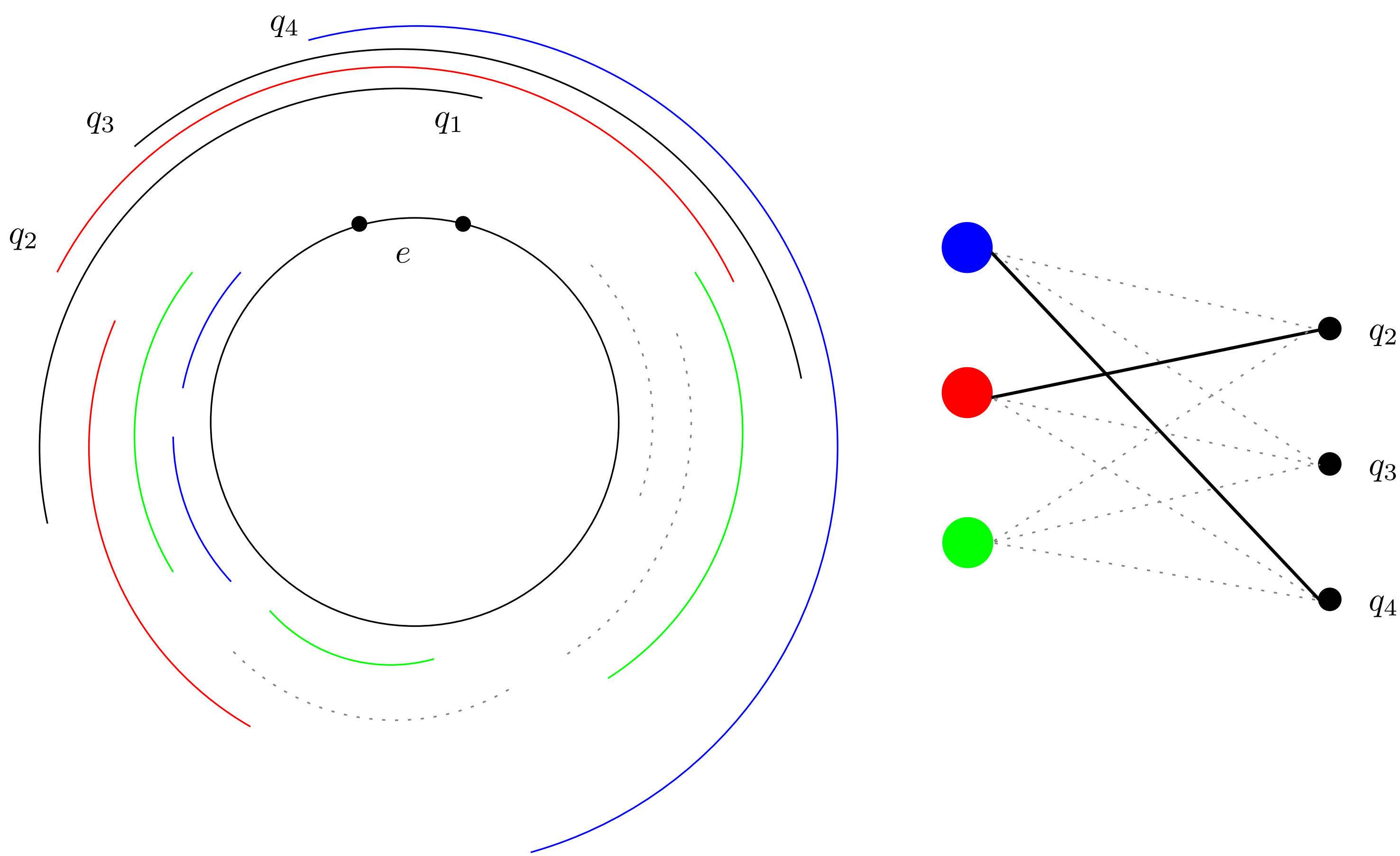
1. pick an edge e, and partition P into P_e and P_c
2. color $\langle G, P_c, k, w\rangle$ optimally (chain subinstance)
3. construct a weighted complete bipartite graph H with nodes $\{1, \dots, k\} \cup K$ (K: set of k heaviest paths in P_e)
   - $w'(i, q) = w(q) - w([P_c(i)]^q)$ (gain by picking $q \in P_e$ instead of $[P_c(i)]^q$)
4. compute a maximum weight matching M in H
5. **for each** $(i, q) \in M$
6. uncolor all paths in $[P_c(i)]^q$ and color q with i

Match and replace (cont'd)





Match and replace (cont'd)



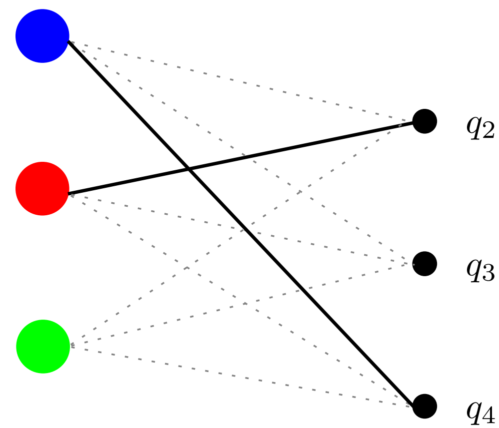
Match and replace (cont'd)

- $\mathrm{OPT} \leq \mathrm{OPT}_e + \mathrm{OPT}_c$
- We need to show that
  - $\mathrm{OPT}_c \leq \mathrm{SOL}$ (easy!)
  - $\mathrm{OPT}_e \leq \mathrm{SOL}$
- These imply $\mathrm{OPT} \leq 2\mathrm{SOL}$
- Remains to prove:

$$\mathrm{OPT}_e = w(K) \leq \mathrm{SOL}$$

Match and replace (cont'd)

The solution returned has total profit



$$\mathrm{SOL} = \mathrm{SOL}_c + w'(M) \ ,$$

which can be written as

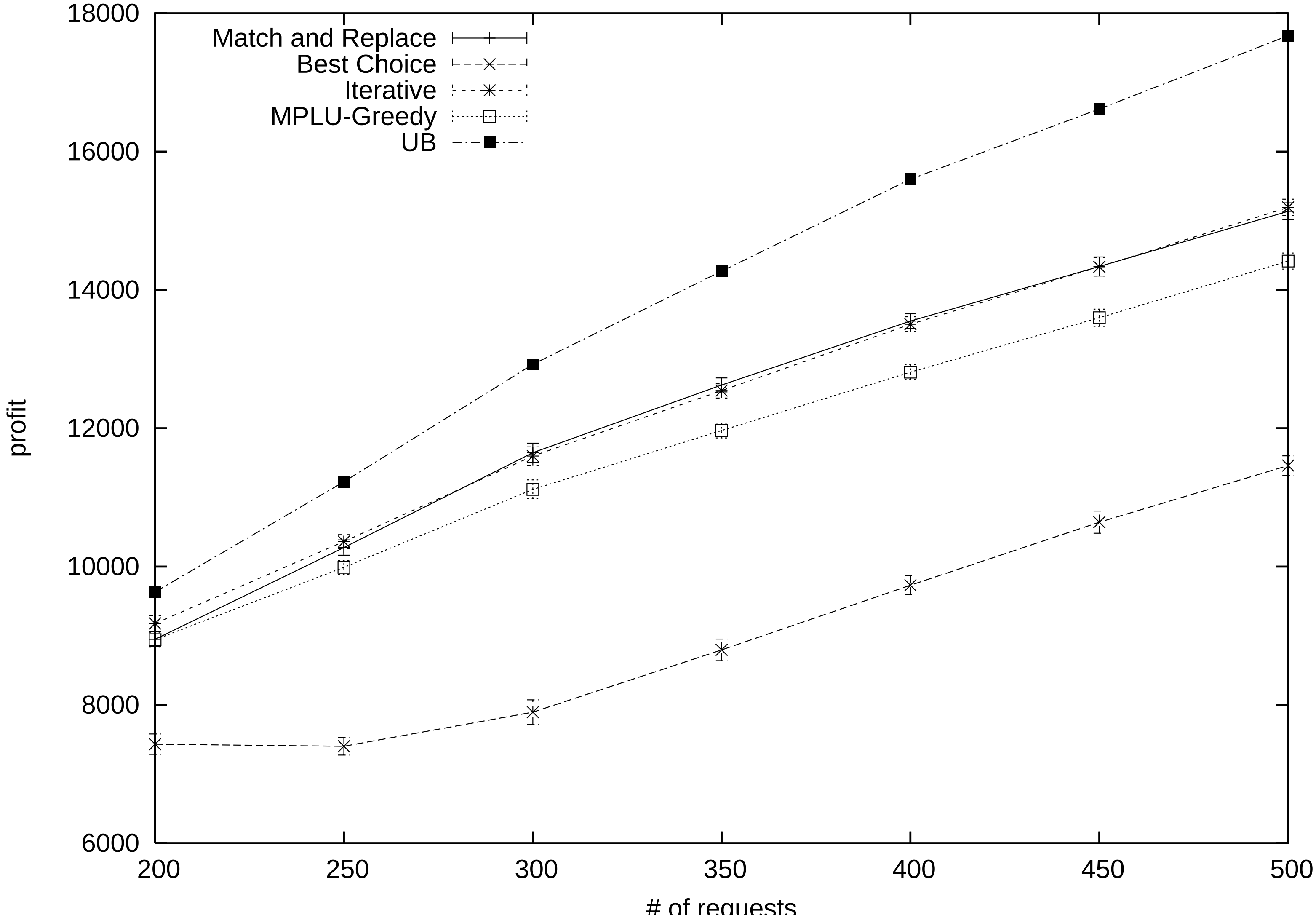
$$\mathrm{SOL} = \sum_{i \text{ not matched}} w(P_c(i)) + w(K_M) + \sum_{(i,q)\in M} w([P_c(i)]^{\neg q})$$

$$\geq \sum_{i \text{ not matched}} w(P_c(i)) - \sum_{q \text{ not matched}} w(q) + w(K) \geq \mathrm{OPT}_e \ .$$

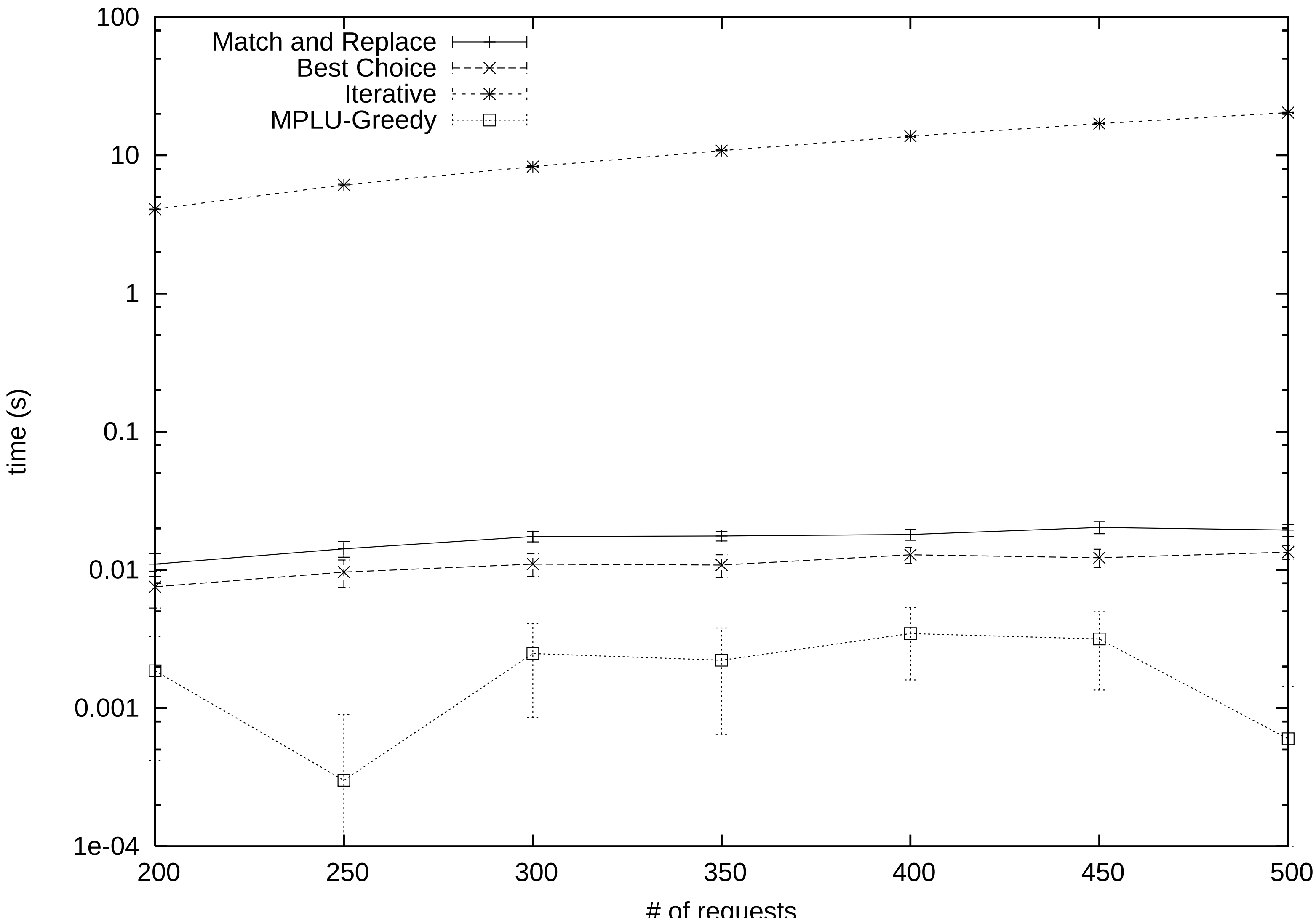
Alternatives for MaxPr-PC in rings

Algorithm	Running time	Appr. guarantee
[Car07]	LP	1.49
Iterative	$O(k^2m^2 \log m)$	1.58
M&R	$O(m^2(k + \log m))$	2
Greedy	$O(nmk + m \log m)$	?
“Best”	$O(km \log m)$	2

profit vs. # paths (n=100, k=80)



time vs. # paths (n=100, k=80)



Further work ([BPPP08])

- Switch model to:
  - non-profit version
  - multifiber setting
  - goal: minimize maximum color multiplicity over all edges
  - paths are non-cooperative, selfish players
- Questions:
  - convergence to Nash Equilibrium (NE)?
  - price of anarchy?
  - price of stability?

Overview of results in selfish model

- Convergence to Nash Equilibrium in at most $4^{|P|}$ steps
- Efficient computation of NE:
  - optimal NE in a subclass of tree games
  - $\frac{1}{2}$-approximate NE in stars
- Upper and lower bounds for the PoA:
  - # colors
  - minimum length of any path that contributes to the cost of some worst-case NE
  - matching lower bounds for graphs with $\Delta \geq 3$
  - constant for a large subclass of ring games

Directions for future work

- Purely combinatorial algorithm with $\rho < 1.49$?
- PTAS for MAXPR-PC ?

In the non-cooperative setting:

- Investigate the case of weighted paths
- Allow players to also choose their routing, and compare with existing models

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[WL98] P.J. Wan, L. Liu: Maximal throughput in wavelength-routed optical networks. Multi-channel Optical Networks: Theory and Practice. Volume 46 of DIMACS Series in Discrete Mathematics and Theoretical Computer Science, AMS (1998) 15-26

[NPZ03] C. Nomikos, A. Pagourtzis, S. Zachos: Satisfying a maximum number of pre-routed requests in all-optical rings. Computer Networks 42(1): 55-63 (2003)

[Car07] I. Caragiannis: Wavelength Management in WDM Rings to Maximize the Number of Connections. STACS 2007: 61-72

[LLWZ05] J. Li, K. Li, L. Wang, H. Zhao: Maximizing Profits of Routing in WDM Networks. J. Comb. Optim. 10(2): 99-111 (2005)

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